4. Mathematics

New Syllabus

Course Contents

Unit 1: Sets, Real Number System and Logic — **10 hrs**

Sets:
Sets and set operations, Theorems based on set operations.

Real Number System:
Real numbers, Field axioms, Order axioms, Interval, Absolute value, Geometrical representation of the real numbers.

Logic:
Introduction, statements, Logical connectives, Truth tables, Basic laws of logic.

Unit 2: Relations, Functions and Graphs — **12 hrs**

Relations:
Ordered pair, Cartesian product, Geometrical representation of Cartesian product, relation, Domain and range of a relation, Inverse of a relation.

Functions:
Definition, Domain and range of a function, Functions defined as mappings, Inverse function, Composite function, functions of special type(Identity, Constant, Absolute value, Greatest integer), Algebraic (Linear, quadratic and cubic), Trigonometric,
Exponential logarithmic functions and their graphs.

Unit 3: Curve Sketching — **10 hrs**

Odd and even functions, Periodicity of a function, symmetry (about x – axis, y– axis and origin) of elementary functions, Monotonocity of a function,
Sketching graphs of polynomial functions $\left(\frac{1}{x}, \frac{x^2-a^2}{x-a}, \frac{1}{x+a}, x^2, x^3\right)$,
Trigonometric, exponential, logarithmic functions (simple cases only)

Unit 4: Trigonometry — **10 hrs**

Inverse circular functions, Trigonometric equations and general values, properties of a triangle (sine law, Cosine law, tangent law, Projection laws, Half angle laws), the area of a triangle. Solution of a triangle (simple cases)

Unit 5: Sequence and Series, and Mathematical Induction — **12 hrs**

Sequence and Series:
Sequence and series, type of sequences and series (Arithmetic, Geometric, Harmonic),
Propertics of Arithmetic, Geometric, and Harmonic sequences, A.M., G.M. And H.M.
Relation among A.M., G.M. and H.M., Sum of infinite geometric series.

Mathematical Induction:
Sum of finite natural numbers, Sum of the squares of first n – natural numbers, Sum of cubes of first n – natural numbers. Intuition and induction, principle of mathematical induction.

Unit 6: Matrices and Determinants — **8 hrs**

Matrices and operation on matrices (Review), Transpose of a matrix and its properties, Minors and Cofactors, Adjoint, Inverse matrix. Determinant of a square matrix, properties of determinants (Without proof) upto 3×3.

Unit 7: System of Linear Equations — **8 hrs**

Consistency of system of linear equations, solution of a system of linear equations by Cramer's rule, Matrix method (row – equivalent and Inverse) upto three variables.

Unit 8: Complex Number — **12 hrs**

Definition of a complex number, Imaginary unit, Algebra of complex numbers, Geometric representation of a complex number, Conjugate and absolute value (Modulus) of a complex numbers and their properties, Square root of a complex number, Polar form of a complex number, product and Quotient of complex numbers.

De Moivre's theorem and its application in finding the roots of a complex number, properties of cube roots of unity.

Unit 9: Polynomial Equations **8 hrs**

Polynomial function and polynomial equations, Fundamental theorem of algebra (without proof), Quadratic equation Nature and roots of a quadratic equation, Relation between roots and coefficients, Formation of a quadratic equation, Symmetric roots, one or both roots common.

Unit 10: Co-ordinate Geometry **12 hrs**

Straight line:

Review of various forms of equation of straight lines, Angle between two straight lines, condition for parallelism and perpendicularity, length of perpendicular from a given point to a given line, Bisectors of the angles between two straight lines.

Pair of lines:

General equation of second degree in x and y, condition for representing a pair of lines, Homogeneous second degree equation in x and y, Angle between pair of lines, Bisectors of the angles between pair of lines.

Unit 11: Circle **10 hrs**

Equation of a circle in various forms (Centre at origin, centre at any point, general equation of a circle, circle with a given diameter), Condition of Tangency of a line at a point to the circle, Tangent and normal to a circle.

Unit 12: Limit and Continuity **10 hrs**

Limits of a function, Indeterminate forms, Algebraic properties of limits (without proof), Theorem on limits of algebraic, Trigonometric, Exponential and logarithmic functions

$$\left(\lim_{x \to a} \frac{x^n - a^n}{x - a}, \lim_{x \to 0} \frac{\sin x}{x}, \lim_{x \to 0} \frac{e^x - 1}{x}, \lim_{x \to 0} \frac{\log(1 + x)}{x}\right)$$

Continuity of a function, Types of discontinuity, Graph of discontinuous function.

Unit 13: The Derivatives **8 hrs**

Derivative of a function, Derivatives of algebraic, trigonometric, exponential and logarithmic functions by definition (simple forms), Rules of differentiation, Derivatives of parametric and implicit functions, Higher order derivatives.

Unit 14: Applications of Derivatives **12 hrs.**

Geometric interpretation of derivative, Monotonocity of a function, Interval of monotonocity, Extrema of a function, Concavity, Points of inflection, Derivative as rate measure.

Unit 15: Antiderivatives and its Applications **10 hrs**

Antiderivative, Integration using basic integrals, Integration by substitution and by parts method, the definite integral, The definite integral as an area under the given curve, Area between two curves.

Model Question

HSEB Examination 2070 (2013)

Time: 3 hrs.

Full Marks:- 100
Pass Marks:- 35

Attempt **all** the questions.

Group 'A' $5 \times 3 \times 2 = 30$

1. (a) Prepare a truth table for the compound statement P∨~(p∧q). What would you conclude from the truth table? **[From unit 1]**

(b) Let A = {1, 2, 3, 4}. Find the relation on A satisfying the condition $x+y \leq 4$. **[From unit 1]**

(Ans: {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)})

(c) Examine whether the function:

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$ is even or odd. Also examine for its symmetricity.

(Ans: f(x) is odd function, symmetric about origin) **[From unit 3]**

2. (a) Prove that : $\tan^{-1}a - \tan^{-1}c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$ [From unit 4.1]

(b) Using principle of mathematical induction, prove that: $1 + 3 + 5 + 7 + \ldots\ldots\ldots\ldots + (2n-1) = n^2$. [From unit 5]

(c) If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find AA^T. [From unit 6]

(Ans: $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$)

3. (a) Using Cramer's rule, solve the following equations: $3x - 2y = 8,\ 5x + 3y = 7$. [From unit 7]

(Ans: x = 2, y = – 1)

(b) If $\alpha = \frac{1}{2}(-1+\sqrt{-3}),\ \beta = \frac{1}{2}(-1-\sqrt{-3})$,

Show that : $\alpha^4 + \alpha^2\beta^2 + \beta^4 = 0$. [From unit 8]

(c) If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k.

(Ans: 1, 4) [From unit 9]

4. (a) Find the distance between the two parallel lines. $3x + 5y = 11$ and $3x + 5y = -23$ [From unit 10.1]

(Ans: $\sqrt{34}$)

(b) Find the equation of the circle whose two of the diameters are $x + y = 6$ and $x + 2y = 8$ and radius 10. [From unit 11]

(Ans: $x^2 + y^2 - 8x - 4y - 80 = 0$)

(c) Evaluate : $\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16}$ [From unit 12.1]

(Ans: 6)

5. (a) Find $\frac{dy}{dx}$ when $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$. [From unit 13]

(Ans: $\frac{t^2+1}{t^2-1}$)

(b) Evaluate: $\int \frac{1}{\sqrt{2x+1} - \sqrt{2x-3}}\,dx$. [From unit 15.1]

(Ans: $\frac{1}{12}[(2x+1)^{3/2} + (2x-3)^{3/2}]$)

(c) Find the interval in which the function $f(x) = 3x^2 - 6x + 5$ is increasing or decreasing. [From unit 14.1]

(Ans: Increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$)

Group 'B' 5×2×4=40

6. (a) Define union and intersection of two sets. If A, B and C are any three non–empty sets, prove that:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [From unit 1]

Or

Let $A = [-3, 1]$ and $B = [-2, 4]$. Find $A \cup B$, $A \cap B$, $A - B$ and $B - A$. [From unit 1]

(Ans: $A \cup B = [-3, 4], A \cap B = [-2, 1], A - B = [-3, -2), B - A = (1, 4]$)

(b) Using different characteristics, sketch the graph of $y = -x^2 + 4x - 3$. [From unit 3]

7. (a) Solve : $\sin x + \cos x = \sqrt{2}$ $(-2\pi \le x \le 2\pi)$. [From unit 4.2]

(Ans: $-\frac{7\pi}{4}, \frac{\pi}{4}$)

Or

State sine law. Using sine law, prove that:

$\tan \frac{1}{2}(C-A) = \frac{c-a}{c+a} \cdot \cot \frac{B}{2}$ **[From unit 4.3]**

(b) Prove that: $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ **[From unit 6]**

8. (a) Using row equivalent matrix method **or** inverse matrix method, solve the following equations.

$x + 4y + z = 18, 3x + 3y - 2z = 2, -4y + z = -7$ **[From unit 7]**

(Ans: x = 1, y = 3, z = 5)

(b) Find the condition under which the two quadratic equations $ax^2 + bx + c = 0$ and $a^1x^2 + b^1x + c^1 = 0$ may have one root common.

(Ans: $(ab' - a'b)(bc' - b'c) = (a'c - ac')^2$) **[From unit 9]**

9. (a) Find the equations of the tangent and normal to the circle $x^2 + y^2 - 3x + 10y - 5 = 0$ at the point (4, –11) **[From unit 11]**

(Ans: $5x - 12y - 132 = 0, 12x + 5y + 7 = 0$)

(b) Evaluate: $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$ **[From unit 12.1]**

$\left(\textbf{Ans: } \cot\theta + \frac{\theta}{\sin^2\theta}\right)$

Or

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 3+2x & \text{for } -\frac{3}{2} \le x < 0 \\ 3-2x & \text{for } 0 \le x < \frac{3}{2} \\ -3-2x & \text{for } x \ge \frac{3}{2} \end{cases}$$

Show that f(x) is continuous at x = 0 but discontinuous at $x = \frac{3}{2}$. **[From unit 12.2]**

10. (a) Find from first principles the derivative of $\sqrt{2-3x}$.

$\left(\textbf{Ans} - \frac{3}{2\sqrt{2-3x}}\right)$ **[From unit 13]**

(b) Find the area bounded by y-axis, the curve $x^2 = 4a(y - 2a)$ and $y = 6a$.

$\left(\textbf{Ans: } \frac{32}{3}a^2\right)$ **[From unit 15.2]**

Group 'C' 5×6=30

11. Let a function f: A → B be defined by $f(x) = \frac{x+1}{2x-1}$. Find the range of f. Is the function f one to one and onto both? If not, how can the function be made one to one and onto both? **[From unit 2]**

$\left(\text{Ans: } R - \left\{\frac{1}{2}\right\}; \text{ one to one only}\right)$

12. Show that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the following relations.

a) $(G.M.)^2 = A.M. \times H.M.$ b) A.M. > G.M. > H.M. **[From unit 5]**

13. Find the length of the perpendicular drown from the point (x^1, y^1) on the line whose equation is Ax + By + c = 0. **[From unit 10.1]**

$$\left(\textbf{Ans: } \left|\frac{Ax' + By' + C}{\sqrt{A^2 + B^2}}\right|\right)$$

Or

Find the equation to the pair of lines joining the origin to the inter section of the straight line y = mx + c and the curve $x^2 + y^2 = a^2$. Prove that they are at right angles if $2c^2 = a^2(1 + m^2)$. **[From unit 10.2]**

(Ans: $(c^2 - a^2m^2)x^2 + 2a^2m\,xy + (c^2 - a^2)y^2 = 0$)

14. Find the square root of the complex number –5 + 12i. **[From unit 8]**

(Ans: ± (2 + 3i))

15. List the criteria for the function y = f(x) to have the local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection. **[From unit 14.2]**

$$\left(\textbf{Ans: Min: 3 at x= 2 Max : 9.75 at } x = \frac{1}{2}, \text{ Point of inflection } x = \frac{5}{4}\right)$$

Or

A spherical ball of salt is dissolving in water in such a way that the rate of decrease in volume at any instant is proportional to the surface. Prove that the radius is decreasing at the constant rate. **[From unit 14.3]**

HSEB Questions

Unit 1 – Sets, Real Number System and Logic

1. If A = {a, e, i}, B = {e, u), U = {a, e, i, o, u}, find $\overline{A \cup B}$ and $\overline{A \cap B}$. **[Q.N.1(a), 2056]**

(Ans: {0} and {a, i, o, u})

2. Rewrite, without using absolute value sign for |3x + 2| <1. **[Q.N.1(b), 2056]**

$$\left(\textbf{Ans: } -1 \le x \le \frac{-1}{3}\right)$$

3. Define the union and the intersection of two sets. If A , B and C are subsets of U, prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **[Q.N.7(a), 2056]**

4. Prove that for any positive real number a, |x|< a implies – a < x < a. **[Q.N.1(a), 2057]**

5. Define the complement of a set. State and prove De-Morgan's laws. **[Q.N.7(a), 2057]**

6. If U = {1,2,......10), A = {x : x ≥ 4}, B = {x : x < 8}, find A ∩ B & A – B. **[Q.N.1(a), 2058]**

(Ans: {4, 5, 6, 7} and {8, 9, 10})

7. If A, B, C are subsets of a universal set U, prove that, A – (B ∩ C) = (A – B) ∪ (A – C) **[Q.N.7(a), 2058]**

8. If A = {a, e, i, o u} and B = {a, b, c, d, e} are two given sets, find A ∪ B and A – B. **[Q.N.1(a), 2059]**

(Ans: {a, b, c, d, e, i, o, u} and {i, o, u})

9. In a certain village in Nepal, all the people speak Nepali or Tharu or both languages. If 90% speak Nepali and 20% Tharu language, how many speak (i) Nepali only (ii) Tharu language only and (iii) both languages. **[Q.N.7(a), 2059]**

Ans:
- **Persons speaking Nepali only = 80%**
- **Persons speaking Tharu only = 10%**
- **Persons speaking both languages = 10%**

10. If O = {1, 3, 5, 7, 9} and P = {2, 3, 5, 7}, find O ∩ P and P – O with the help of Venn diagram. **[Q.N.1(a), 2060]**

(Ans: O ∩ P ={3, 5, 7} and P – O = {2})

11. In a group of students 24 study Maths, 30 study Biology, 22 study physics; 8 study Maths only, 14 study Biology only, 6 study Biology and Physics only, and 2 study Maths and Biology only.
Find :
(i) how many study all three subjects.
(ii) how many students were in the group. **[Q.N.7(a), 2060]**
(Ans: (i) 8 (ii) 46)

12. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} be the universal set and M = {1, 3, 5, 7}, N = {2, 4, 6, 8}, then find $\overline{M \cup N}$ and $\overline{M \cap N}$. **[Q.N.1(a), 2061]**
(Ans: $\overline{M \cup N}$ = {9} and $\overline{M \cap N}$ = {1,2,3,4,5,6,7,8,9})

13. In a group of twenty eight teachers of a school, 15 teach English, 15 teach Maths, 14 teach Nepali, 7 teach English and Maths, 6 teach English and Nepali, 5 teach Maths and Nepali. Find how many teach all three subjects, how many teach Maths only and Nepali only. **[Q.N.7(a), 2061]**
(Ans: $n(E \cap M \cap N) = 2$, $n_0(M) = 5$ and $n_0(N) = 5$)

14. If A = {1, 3, 5, 7} & B = {2, 3, 5}, find $A \cap B$ and A – B. Show them in Venn-diagram. **[Q.N.1(a), 2062]**
(Ans: $A \cap B$ = {3, 5} and A – B = {1, 7})

15. Define Union and Intersection of two sets. Illustrate them through Venn-diagrams. Let A, B, C be any non-empty subsets of U, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **[Q.N.7(a), 2062]**

16. Find $A \cap B$ if:
$A = \{x : x = 2n+1, n \le 6, n \in N\}$, $B = \{x : x = 3n-2, n \le 3, n \in N\}$ **[Q.N.1(a), 2063]**
(Ans: {7})

17. Write the following by using the modulus sign : $-1 \le x \le 5$
(Ans: $|x - 2| \le 3$) **[Q.N.3(c), 2063]**

18. Of the number of three athletic teams, 21 are in the basketball team, 26 in hockey team and 29 in football team, 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball, and 8 play all the games. How many members are there in all ? **[Q.N.7(a), 2063]**
(Ans: 43)

19. Find $A \cup B$ if :
$A = \{x : x = 2n+1, n \le 5, n \in N\}$
$B = \{x : x = 3n-2, n \le 4, n \in N\}$ **[Q.N. 1(a), 2064]**
(Ans: {1, 3, 4, 5, 7, 9, 10, 11})

20. Write $| x - 7 | < 3$ without using the modulus sign. **[Q.N. 3(c), 2064]**
(Ans: $4 < x < 10$)

21. A village has total population 25,000 out of which 13,000 read 'Gorkhaptra' and 10,500 read 'Kantipur' and 2500 read both papers. Find the percentage of population who read neither of these papers. **[Q.N. 7(a), 2064]**
(Ans: 16%)

22. Given A = [–2, 4) and B = (2, 5], compute $A \cup B$ and $A \cap B$. **[Q.N. 1(a), 2065]**
(Ans: [–2,5], [2,4))

23. Out of a group of 20 teachers in a school, 10 teach Maths, 9 teach Physics, 7 teach Chemistry, 4 teach Maths and Physics, but none teach both Maths and Chemistry :
(i) How many teach Physics and Chemistry ?
(ii) How many teach only Physics ?
(iii) How many teach only Chemistry ? **[Q.N. 7(a), 2065]**
(Ans: (i) 2 (ii) 3 (iii) 5)

24. Given A = {1, 2, 3} and B = {3, 4, 5, 6}, show that $A - (A - B) = A \cap B$. **[Q.N.1 (a), 2066]**

25. Twenty three medals are awarded for folksongs, eight for Deuda songs and eleven for Maithili songs. If the total number of singers awarded is thirty two and only three singers received medals in all three types of songs, find how many singers received medals in exactly two of three types of songs.
(Ans: 4) **[Q.N.7 (a), 2066]**

26. Define Power set. Write the power set of the set A {a, b, c}.
(Ans: {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}} **2 [Q.N. 1 (a), 2067]**

27. In a group of students 30 study maths, 24 study physics, 22 study chemistry, 14 study maths only, 8 study physics only, 6 study maths and chemistry only, 2 study maths and physics only and 8 study none. How many students are in the group ? How many study chemistry only ? How many study all three subjects ?
(Ans: 60, 2, 8) **4 [Q.N.7 (a), 2067]**

28. Define disjunction of two statements. Prepare a truth table for the compound statement $\sim (p \vee q)$ **[Q.N.1(a),2068]**

29. Let A = {1,2,3,4} and B = {1,3,5}. Find the relation R from Set A to Set B determined by the condition x > y.
(Ans: {(2, 1), (3, 1), (4, 1), (4, 3)} **[Q.N.1(b),2068]**

30. If A, B and C be any three non-empty sets,
Prove that: $A - (B \cup C) = (A - B) \cap (A - C)$. **[Q.N.6(a),2068]**

31. Define absolute value of a real number. Rewrite the following relation without using absolute value sign
$|2x - 1| \leq 5$.
Also, draw the graph of the inequality. **[Q.N.6(a)(Or),2068]**

32. Define negation of a statement. Construct a truth table for the compound statement $\sim(p \vee (\sim q))$. **[Q.N. 1(a), Set 'A' 2069]**

33. Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
[Q.N. 6(a), Set 'A' 2069]

34. If $x \in R$ and a is any positive real number,
Prove that: $|x| < a \Rightarrow -a < x < a$ and conversely. **[Q.N. 6(a)(Or), Set 'A' 2069]**

35. Write inverse and converse of the statement 'if 3 is an odd number then 6 is not an odd number'. **[Q.N. 1(a), Set 'B' 2069]**

36. Define De–Morgan's law. For any non-empty sets, A, B, C prove:
$A - (B \cup C) = (A - B) \cap (A - C)$. **[Q.N. 6(a), Set 'B' 2069]**

37. Define absolute value of a real number. For any two real numbers. x and y, prove that: $|x + y| \leq |x| + |y|$. **[Q.N. 6(a)(Or), Set 'B' 2069]**

38. Define disjunction of two functions. Construct a truth table for the compound statement. $(\sim p) \vee (\sim q)$. **[Q.N. 1(a), Supp. 2069]**

39. Let A = {a, b}, B = {b, c} and C = {c, d}. Find $A \times (B \cup C)$ and $A \times (B \cap C)$.
[Q.N. 1(b), Supp. 2069]
(Ans: {(a, b), (a, c), (a, d), (b, b), (b, c), (b, d)}, {(a, c), (b, c)})

40. If A, B and C are any three non–empty sets, prove that:
$A - (B \cap C) = (A - B) \cup (A - C)$ **[Q.N. 6(a), Supp. 2069]**

41. Prepare a truth table for the compound statement $P \vee \sim (p \wedge q)$.
What would you conclude from the truth table? **[Q.N. 1(a), 2070 'C']**

42. Let A = {1, 2, 3, 4}. Find the relation on Asatisfying the condition $x+y \leq 4$.
(Ans: {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}) **[Q.N. 1(b), 2070 'C']**

43. Define union and intersection of two sets. If A, B and C are any three non–empty sets, prove that:
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **[Q.N. 6(a), 2070 'C']**
(Ans: $A \cup B = [-3, 4]$, $A \cap B = [-2, 1]$, $A - B = [-3, -2)$, $B - A = (1, 4]$)

44. Let A = [–3, 1] and B = [–2, 4]. Find $A \cup B$, $A \cap B$, A – B and B – A.
{Ans: [– 3, 4]; [– 2, 1], [– 3, – 4), (1, 4]} **[Q.N. 6(a)(Or), 2070 'C']**

45. If p and q are any two statements, prove that: $p \vee q \equiv q \vee p$. [Q.N. 1(a), 2070 'D']

46. Let A = {a, b}, B = {b, c} and C = {c, d}. Find :
$A \times (B \cup C)$ and $A \times (B \cap C)$. [Q.N. 1(b), 2070 'D']
(Ans: {(a, b), (a, c), (a, d), (b, b), (b, c), (b, d)}, {(a, c), (b, c)})

47. Define union and intersection of two sets. If A, B and C are any three non–empty sets, prove that:
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [Q.N. 6(a), 2070 'D']

Unit 2 – Relations, Functions and Graphs

1. Let f : R→R and g: R→R be defined by $f(x) = x^2+1$; $g(x) = x^5$
find f^{-1}; (gof) (x) and (fog) (x). [Q.N.7(b), 2056]
(Ans: $(x-1)^{1/2}$, $(x^2+1)^5$ and $x^{10}+1$)

2. When does a function $f : A \to B$ become an onto and one to one ?
[Q.N.1(b), 2057]

3. A function f (x) is defined as follows :
$$f(x) \begin{matrix} = 4x - 2 \text{ for } x \geq 1 \\ = 2x \quad \text{ for } x \angle 1 \end{matrix} \Bigg\}, \text{ find } f(2)\,;\, f(1)\,;\, f(0)\,;\, f(-1)\,;\, \frac{f(h)-f(1)}{h} \text{ for } 1 \leq h.$$
[Q.N.7(b), 2057]
$$\left(\textbf{Ans: } \begin{matrix} f(2) = 6, f(1) = 2, f(0) = 0, \\ f(-1) = -2 \text{ and } \dfrac{f(h)-f(1)}{h} = \dfrac{4(h-1)}{h} \end{matrix}\right)$$

4. Let $f: A \to R$ be given by $f(x) = 2|x| + 3$
Where A = {-2,0,1,2,}, find the range of f. [Q.N.1(b), 2058]
(Ans: {3, 5, 7})

5. Check whether the function $f = [-2, 3] \to R$ given by $f(x) = x^3$ is one to one, onto or both. [Q.N.7(b), 2058]
(Ans: One to one but not onto)

6. Let f: R→R be defined by $y = f(x) = 2x - 3$, $x \in R$, find formula that defines f^{-1}
[Q.N.1(c), 2059]
$$\left(\textbf{Ans: } f^{-1}(x) = \frac{x+3}{2}\right)$$

7. Let f: R→R and g: R→R be defined by $f(x) = x^2 + 1$ and $g(x) = x^5$, find f^{-1}, (gof) (x); (fog) (x). [Q.N.7(b), 2059]
$$\left(\textbf{Ans: } \begin{cases} f^{-1}(x) = (x-1)^{\frac{1}{2}} \\ (gof)(x) = (x^2+1)^5 \\ \text{and } (fog)(x) = x^{10}+1 \end{cases}\right)$$

8. Define even add odd functions with examples. [Q.N.1(c), 2060]

9. Let $f : R \to R$ is defined by $f(x) = 2x + 3$ and
$g : R \to R$ is defined by $g(x) = x^2$, find $(g_o f)x$ and $(f_o g)x$. [Q.N.7(b), 2060]
(Ans: (gof) (x) = $(2x+3)^2$ and (fog) (x) = $(2x^2+3)$)

10. Let $h : R \to R$ be defined by $h(x) = 3x - 6$, find a formula that defines h^{-1}.
[Q.N.1(b), 2061]
$$\left(\textbf{Ans: } h^{-1}(x) = \frac{x-6}{3}\right)$$

11. For the functions $f(x) = 2x^2 - 3$ and $g(x) = 3x + 2$ where $x \in \mathbb{R}$, determine $fog(x)$, $gof(x)$. Are (fog) and (gof) one–one ? [Q.N.7(b), 2061]
(Ans: Not one-one)

12. Check whether the function $f : [-2,3] \to R$ given by $f(x) = x^3$ is one to one, onto or both. [Q.N.1(b), 2062]
(Ans: f is one to one but not onto)

13. When does an inverse of a function exists ? If $f : R \to R$ be defined by $f(x) = x^3 + 5$, find the formula that defines f^{-1}. [Q.N.7(b), 2062]
(Ans: $f^{-1}(x) = \sqrt[3]{x-5}$)

14. Let f, g be real valued function defined as
$f(x) = x^2 + 5x + 7$, $x \in R$ and
$g(x) = 5x - 3$, $x \in R$
find $fog(x)$ and $gof(x)$. [Q.N.1(b), 2063]
(Ans: $(25x^2 - 5x + 1)$ & $(5x^2 + 25x + 32)$)

15. Let R be the set of rational numbers. Show that the function defined by $f : R \to R$ such that $f(x) = 4x - 7$
$x \in R$ is one-one and onto. Find a formula for f^{-1}. [Q.N.7(b), 2063]
$\left(\text{Ans: } f^{-1}(x) = \frac{x+7}{4}\right)$

16. Let f, g be real valued functions defined as :
$f(x) = 4x + 7$, $x \in R$ and
$g(x) = 5x - 2$, $x \in R$
find $fog(x)$ and $gof(x)$ [Q.N. 1(b), 2064]
(Ans: $(20x - 1)$ $(20x + 33)$)

17. Let Q be the set of all rational numbers. Show that the function:
$f: Q \to Q$ such that $f(x) = 3x + 5$ for all $x \in Q$ is one to one and onto. Find f^{-1}.
$\left(\text{Ans: } \left[f^{-1}(x) = \frac{x-5}{3}\right]\right)$ [Q.N. 7(b), 2064]

18. Let $f : R \to R$ be defined by $f(x) = 2x + 5$. Compute $f^{-1}(x)$. [Q.N. 1(b), 2065]
$\left(\text{Ans: } f^{-1}(x) = \frac{x-5}{2}\right)$

19. If f, g : R→R defined by, $f(x) = x^3 + 2$ and $g(x) = 4x - 1$, find $fog(x)$ and $gof(x)$, and show that the composite function is not commutative. [Q.N. 7(b), 2065]

20. Prove that $\log_a x^2 - 2 \log_a \sqrt{x} = \log_a x$ [Q.N.1 (b), 2066]

21. Let R be the set of real numbers. Show that the function $F: R \to R$ such that $f(x) = 5x - 3$ for all $x \in R$ is one to one and onto. [Q.N.7 (b), 2066]

22. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, prove that $xyz = 1$. 2 [Q.N. 1 (b), 2067]

23. For the function $f(x) = 2x^2 - 3$ and $g(x) = 3x + 2$, $x \in R$ examine whether fog and gof are one-one.
(Ans: both are not one-one) 4 [Q.N.7 (b), 2067]

24. Define the domain and the range of a function. Find the domain and the range of the function $f(x) = -x^2 + 4x - 3$. [Q.N.11,2068]
(Ans: Domain = R = $(-\infty, \infty)$, Range = $(-\infty, 1]$)

25. Find the domain of the function $y = \sqrt{x-2}$.
[Ans: 2, ∞] [Q.N. 1(b), Set 'A' 2069]

26. Draw the graph of the function $y = x^2 - 4x + 3$ using its different characteristics. [Q.N. 6(b), Set 'A' 2069]

27. Define one to one function and onto function. Let a function $f : A \rightarrow B$ be defined by $f(x) = \frac{x^2}{6}$ with $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, \frac{1}{6}, \frac{2}{3}\}$.
Find the range of f. Is the function f one to one and onto both?
$\left(\text{Ans: } 0, \frac{1}{6}, \frac{2}{6} \text{ and onto only}\right)$ [Q.N. 11, Set 'A' 2069]

28. Find the domain, range and inverse of the relation R = {(1, 2), (2, 4), (3, 6), (4, 8)} [Q.N. 1(b), Set 'B' 2069]
(Ans: {1, 2, 3, 4}, { 2, 4, 6, 8}, {(2, 1), (4, 2), (6, 3), (8, 4)}

29. Define function. State the condition for a function to be bijective. Given $f(x) = x^3 + 5$, $x \in R$, find f^{-1} [Q.N. 11, Set 'B' 2069]
(Ans: $f^{-1}(x) = \sqrt[3]{x - 5}$)

30. Solve the inequality $|2x + 1| \geq 3$ and draw its graph. [Q.N. 6(a) (Or), Supp. 2069]

31. Define composite function of $f = R \rightarrow R$ and $g = R \rightarrow R$ be defined by $f(x) = 3x^2 - 4$ and $g(x) = 2x - 5$.
find (gof) (x), (fog) (x) and (fof) (x). Is (gof) x = (fog) x? Is the composite function (gof) (x) one to one? Give reason. [Q.N. 11, Supp. 2069]
(Ans: $6x^2 - 13$, $12x^2 - 60x + 71$, $3x^4 - 24x^2 + 44$, not one to one)

32. Let a function $f: A \rightarrow B$ be defined by $f(x) = \frac{x+1}{2x-1}$. Find the range of f. Is the function f one to one and onto both? If not, how can the function be made one to one and onto both? [Q.N. 11, 2070 'C']
$\left(\text{Ans: } R - \left\{\frac{1}{2}\right\}; \text{ one to one only}\right)$

33. Solve the inequality : $6 + 5x - x^2 \geq 0$. [Q.N. 6(a)(Or), 2070 'D']
(Ans: $\{x : -2 \leq x \leq 3\}$)

34. Define composite function of two functions f and g. Let $f:R \rightarrow R$ and $g:R \rightarrow R$ be two functions defined by $f(x) = 2x^2 - 3$ and $g(x) = 3x + 2$. Determine (fog) (x), (gof) (x) and (gog) (x). Is (fog) (x) = (gof) (x) ? Are the functions (fog) (x) and (gog) (x) one to one? [Q.N. 11, 2070 'D']
(Ans: $18x^2 + 24x + 5$, $6x^2 - 7$, $9x + 8$, not one to one))

Unit 3 – Curve Sketching

1. Test the periodicity and the symmetricity of the function y = sin x. [Q.N.1(c),2068]

2. Draw the graph of $y = \cos x \left(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right)$ using its different characteristics. [Q.N.6(b),2068]

3. Test the periodicity of the function $f(x) = \sin 2x$ and find its period. [Q.N. 1(c), Set 'A' 2069]

4. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, show that:
$x^2 + y^2 + z^2 + 2xyz = 1$ [Q.N. 7(a), Set 'A' 2069]

5. State sine law. Prove that: $\tan \frac{1}{2}(B-C) = \frac{b-c}{b+c} \cot \frac{A}{2}$. [Q.N. 7(a)(Or), Set 'A' 2069]

6. Examine the function y = cosx for symmetry and even or odd nature. [Q.N. 1(c), Set 'B' 2069]

7. Sketch the graph of $f(x) = (x - 4)^2 - 8$ indicating its characteristics. [Q.N. 6(b), Set 'B' 2069]

8 Test the periodicity of the function $f(x) = \cos \pi x$ and find its period.
(Ans: 2) **[Q.N. 1(c), Supp. 2069]**

9. Draw the graph of the function $f(x) = x^2 - 2x - 3$ with its different characteristics. **[Q.N. 6(b), Supp. 2069]**

10. Examine whether the function:
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is even or odd. Also examine for its symmetricity.
(Ans: f(x) is odd function, symmetric about origin) **[Q.N. 1(c), 2070 'C']**

11. Using different characteristics, sketch the graph of $y = -x^2 + 4x - 3$. **[Q.N. 6(b), 2070 'C']**

12. Test the even or odd nature and the symmetricity of the function $f(x) = x^4 + 3x^2 + 1$. **[Q.N. 1(c), 2070 'D']**
(Ans: even, symmetric about y-axis)

13. Using different characteristics, sketch the graph of $y = (x - 1)(x - 2)(x - 3)$. **[Q.N. 6(b), 2070 'D']**

Unit 4 – Trigonometry

4.1 Inverse Circular Function

1. Find the value of $\tan^{-1}3 + \tan^{-1}\frac{1}{3}$. **[Q.N.2(a), 2056]**
$\left(\text{Ans: } \frac{\pi}{2}\right)$

2. Solve: $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1}x$. **[Q.N.8(a), 2056]**
$\left(\text{Ans: } x = \frac{a+b}{1-ab}\right)$

3. Prove that : $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$ **[Q.N.2(a), 2057]**

4. Find the value of $\cos \tan^{-1} \sin \cot^{-1} x$. **[Q.N.8(a) (Or), 2057]**
$\left(\text{Ans: } \sqrt{\frac{(1+x^2)}{(2+x^2)}}\right)$

5. Find the value of $\tan^{-1}3 + \tan^{-1}\frac{1}{3}$ **[Q.N.2(a), 2058]**
$\left(\text{Ans: } \frac{\pi}{2}\right)$

6. Prove that $\cot^{-1}3 + \text{cosec}^{-1}\sqrt{5} = \frac{\pi}{4}$ **[Q.N.8(a), 2058]**

7. Show that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{6}{17}$. **[Q.N.2(a), 2059]**

8. Prove: $\tan(2\tan^{-1}x) = 2\tan(\tan^{-1}x + \tan^{-1}x^3)$. **[Q.N.8(a), 2059]**

9. Show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$. **[Q.N.2(a), 2060]**

10. Prove that $\cot^{-1}(\tan 2x) + \cot^{-1}(\tan 3x) = x$. **[Q.N.8(a), 2060]**

11. Solve : $2\tan^{-1}x = \sin^{-1}\frac{2m}{1+m^2} + \sin^{-1}\frac{2n}{1+n^2}$ **[Q.N.2(a), 2061]**
$\left(\text{Ans: } x = \frac{m+n}{1-mn}\right)$

12. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, prove that $x + y + z = xyz$. **[Q.N.8(a), 2061]**

13. Solve : $Cos(Sin^{-1}x) = \frac{1}{2}$ [Q.N.2(a), 2062]

$\left(\text{Ans: } x = \frac{\sqrt{3}}{2}\right)$

14. Solve : $Tan^2x = Secx+1$ [Q.N.8(a) (Or), 2062]

$\left(\text{Ans: } (6n\pi \pm 1)\frac{\pi}{3} \text{ or } (2n+1)\pi\right)$

15. Prove that: $\tan^{-1}\left(\frac{Sinx}{1+Cosx}\right) = \frac{x}{2}$ [Q.N.2(a), 2063]

16. Prove that :
$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = 2(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3})$. [Q.N.8(a) Or, 2063]

17. Prove that :
$\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) = 0$ [Q.N. 2(a), 2064]

18. Prove that : $Sec^{-1}x + Cosec^{-1}x = \frac{\pi}{2}$ [Q.N. 2(a), 2065]

19. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x + y + z = xyz$. [Q.N. 8(a), 2065]

20. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}x$ [Q.N.2 (b), 2066]

21. If $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \pi$ show that $xy + yx + zx = 1$ [Q.N.8 (a), 2066]

22. Show that $\tan^{-1}x = \frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}$ 2 [Q.N. 2 (a), 2067]

23. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that
$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$. 4 [Q.N.8 (a), 2067]

24. Prove that:
$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi = 2(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3})$
[Q.N.7(a),2068]

25. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ show that :
$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$. [Q.N. 7(a), Set 'B' 2069]

26. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ prove that:
$xy + yz + zx = 1$. [Q.N. 7(a), Supp. 2069]

27. Prove that : $\tan^{-1}a - \tan^{-1}c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$ [Q.N. 2(a), 2070 'C']

28. Prove that : $\cos(\sin^{-1}u + \cos^{-1}v) = v\sqrt{1-u^2} - u\sqrt{1-v^2}$ [Q.N. 2(a), 2070 'D']

4.2 General Values

1. Solve $Sinx + \sqrt{3}\cos x = \sqrt{2}$ [Q.N.8(a) (Or), 2056]

$\left(\text{Ans: } x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}\right)$

2. Solve : $2Sin\,3x - 2\,Sinx + 5\,Cos\,2x = 0$ [Q.N.8(a), 2057]
(Ans: sinx = 45°, 135°, 225°, 315° etc.)

3. Solve: $\tan^2x = \sec x + 1$ [Q.N.8(a) (Or), 2058]

$\left(\text{Ans: } (6n\pi \pm 1)\frac{\pi}{3} \text{ or } (2n+1)\pi\right)$

4. Solve: cot x + tan x = 2. [Q.N.8(a) (Or), 2059]

$\left(\textbf{Ans: } x = n\pi + \frac{\pi}{4}\right)$

5. Solve for general values of x : 7 Sin^2 x + 3 Cos^2 x = 4. [Q.N.8(a) (Or), 2060]

$\left(\textbf{Ans: } x = n\pi \pm \frac{\pi}{6}\right)$

6. Solve for general values of x : cos x + cos 2x + cos3x = 0 [Q.N.8(a) (Or), 2061]

$\left(\textbf{Ans: } \begin{array}{l} x = (2n+1)\frac{\pi}{4} \\ \text{and } x = (6n \pm 2)\frac{\pi}{3} \end{array}\right)$

7. Solve : $Cos3x + Cos2x = Sin\,\frac{3}{2}x + Sin\,\frac{x}{2},\ 0 \le x \le \pi.$ [Q.N.8(a), 2062]

(Ans: Values of x in the internal [0, π] are $\frac{\pi}{7}, \frac{5\pi}{7}, \pi$ **)**

8. Find the general values of *x*, when
Sin2*x*tan*x* + 1 = Sin2*x* + tan*x* [Q.N.8(a), 2063]

$\left(\textbf{Ans: } (4n+1)\frac{\pi}{4}\right)$

9. Find the general values of x when
Cos x + Sin x = Cos 2x + Sin 2x [Q.N. 8(a), 2064]

$\left(\textbf{Ans: } 2n\pi, (4n+1)\frac{\pi}{6}\right)$

10. If sin 2x = 3 sin 2y, prove that : 2 tan (x–y) = tan (x + y) [Q.N. 8(a)(or), 2064]

11. Solve : Sinθ + Sin2θ + Sin3θ = Cosθ + Cos2θ + Cos3θ [Q.N. 8(a, or), 2065]

$\left(\textbf{Ans: } (6n \pm 2)\frac{\pi}{3}, (4n+1)\frac{\pi}{8}\right)$

12. Solve for general values of θ : tan (θ + α). tan (θ – α) = 1.

$\left(\textbf{Ans: } (2n+1)\frac{\pi}{4}\right)$ [Q.N.8 (a) (or), 2066]

13. Solve for general values of x : 2 sin^2x + sin^2 2x = 2 4 [Q.N.8 (a) (Or), 2067]

$\left(\textbf{Ans: } x = \left(2n\pi \pm \frac{\pi}{2}\right), x = \left(n\pi \pm \frac{\pi}{4}\right)\right)$

14. Solve: cotx + tanx = 2 (0 ≤ x ≤ π) [Q.N.2(a),2068]

$\left(\textbf{Ans: } \frac{\pi}{4}\right)$

15. Solve: sinx – cos x = $\sqrt{2}$.

$\left(\textbf{Ans: } x\pi + (-1)^n\frac{\pi}{2} + \frac{\pi}{4}\right)$ [Q.N. 2(a), Set 'A' 2069]

16. Solve: tan2x = tan x (– π ≤ x ≤ π). [Q.N. 2(a), Set 'B' 2069]
(Ans: 0, ± π)

17. Solve: 2cos^2 x + 4sin^2x = 3. [Q.N. 2(a), Supp. 2069]

$\left(\textbf{Ans: } n\pi \pm \frac{\pi}{4}\right)$

18. Solve : sinx + cosx = $\sqrt{2}$ (–2π ≤ x ≤ 2π). [Q.N. 7(a), 2070 'C']

$\left(\textbf{Ans: } -\frac{7\pi}{4}, \frac{\pi}{4}\right)$

19. Solve : $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0.$ **[Q.N. 7(a), 2070 'D']**

$\left(\textbf{Ans: } (6n \pm 1)\frac{\pi}{3}\right)$

4.3 Properties of a Triangle

1. Prove that $r_1r_2 + rr_3 = ab$ **[Q.N.1(c), 2056]**
2. If $a^4+b^4+c^4 = 2c^2(a^2+b^2)$ prove that $\angle C = 45°$ or $135°$ **[Q.N.8(b), 2056]**
3. Prove that : $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$. **[Q.N.1(c), 2057]**
4. If S be the area of incircle and S_1, S_2, S_3 are areas of excircles, show that: $\frac{1}{\sqrt{S_1}} + \frac{1}{\sqrt{S_2}} = \frac{1}{\sqrt{S}} - \frac{1}{\sqrt{S_3}}$ **[Q.N.8(b), 2057]**
5. Prove that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$ **[Q.N.1(c), 2058]**
6. In any ΔABC prove that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ **[Q.N.8(b), 2058]**
7. Prove that: $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$. **[Q.N.1(b), 2059]**
8. In any triangle ABC prove: $\tan\frac{1}{2}(B - C) = \frac{b-c}{b+c}\cot\frac{A}{2}$. **[Q.N.8(b), 2059]**
9. In any triangle ABC, prove that $\sin A + \sin B + \sin C = \frac{s}{R}$ **[Q.N.1(b), 2060]**
10. Prove that, in any triangle ABC, $\frac{b^2 - c^2}{a^2}.\sin 2A + \frac{c^2 - a^2}{b^2}.\sin 2B + \frac{a^2 - b^2}{c^2}.\sin 2C = 0$ **[Q.N.8(b), 2060]**
11. In any triangle ABC, if $\cos B = \frac{\sin A}{2\sin C}$, show that the triangle is isosceles. **[Q.N.1(c), 2061]**
12. In any triangle ABC, prove that $\frac{a^2\sin(B-C)}{\sin B + \sin C} + \frac{b^2\sin(C-A)}{\sin C + \sin A} + \frac{c^2\sin(A-B)}{\sin A + \sin B} = 0$ **[Q.N.8(b), 2061]**
13. Prove that $r\,r_1\,r_2\,r_3 = \Delta^2$ **[Q.N.1(c), 2062]**
14. In any ΔABC, prove that : $\frac{\cos B - \cos C}{\cos A + 1} = \frac{c-b}{a}$ **[Q.N.8(b), 2062]**
15. In a ΔABC, prove that : $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a + b + c$ **[Q.N.1(c), 2063]**
16. In any triangle, prove that $r_1 + r_2 + r_3 - r = 4R$ **[Q.N.8(b), 2063]**
17. In a Δ ABC, prove that $\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$ **[Q.N. 1(c), 2064]**
18. In any triangle, prove that : $a\cos A + b\cos B + c\cos C = 4R\sin A\sin B\sin C$ **[Q.N. 8(b), 2064]**
19. Show that : $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$ **[Q.N. 1(c), 2065]**
20. In any triangle, state and prove Cosine law. **[Q.N. 8(b), 2065]**
21. Show that $\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3} = 0$ **[Q.N.1 (c), 2066]**

22. In a ΔABC, if $(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3\sin A \sin B$ then prove $\angle C = 60°$. **[Q.N.8 (b) (Or), 2066]**

23. In any triangle if $\cos B = \frac{\sin A}{2\sin C}$, prove that the triangle is isosceles. 2 **[Q.N. 1 (c), 2067]**

24. In any triangle ABC if $8R^2 = a^2 + b^2 + c^2$, prove that the triangle is right angled. 4 **[Q.N.8 (b) (Or), 2067]**

25. State Cosine Law, using Cosine Law,
Prove that: $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ **[Q.N.7(a)(Or),2068]**

26. If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{2}{a+b+c}$, show that $c = 60°$. **[Q.N. 7(a) (Or), Supp. 2069]**

27. State sine law. Using sine law, prove that:
$\tan\frac{1}{2}(C-A) = \frac{c-a}{c+a} \cdot \cot\frac{B}{2}$ **[Q.N. 7(a)(Or), 2070 'C']**

28. If $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$, prove that: $C = 45°$ or $135°$. **[Q.N. 7(a)(Or), 2070 'D']**

4.4 Solution of a Triangle

1. If three sides of a triangle are in the ratio $2 : \sqrt{6} : \sqrt{3} + 1$, find the angles. **[Q.N.8(b) (Or), 2056]**
(Ans: A = 45°, B = 60° and C = 75°)

2. Solve the triangle if $a = \sqrt{6}$, $b = 2$ and $c = \sqrt{3} - 1$. **[Q.N.8(b) (Or), 2057]**
(Ans: A = 120°, B = 45°, C = 15°)

3. If $a = 2$, $b = \sqrt{6}$, $c = \sqrt{3} + 1$, solve the triangle. **[Q.N.8(b) (Or), 2058]**
(Ans: A = 45°, B = 60°, C = 75°)

4. Solve the triangle, if $a = 2$, $b = \sqrt{6}$, $c = \sqrt{3} + 1$. **[Q.N.8(b) (Or), 2059]**
(Ans: A = 45°, B = 60° and C = 75°)

5. In any triangle ABC,
$b = \sqrt{3}$, $C = 1$ and $A = 30°$
solve the triangle. **[Q.N.8(b) (Or), 2060]**
(Ans: $a = c = 1$, $b = \sqrt{3}$, $\angle A = \angle C = 30°$, $\angle B = 120°$)

6. In any triangle ABC, if $A = 30°$ and $B = 90°$, find a : b : c **[Q.N.8(b) (Or), 2061]**
$\left(\textbf{Ans: } \frac{1}{2} : 1 : \frac{\sqrt{3}}{2}\right)$.

7. In any ΔABC, $a = 2$, $b = \sqrt{6}$ and $c = \sqrt{3} - 1$, find $\angle B$ **[Q.N.8(b) (Or), 2062]**
(Ans: $\angle B = 60°$)

8. If the angles of a triangle are to one another as 1 : 2 : 3, prove that the corresponding sides are $1 : \sqrt{3} : 2$. **[Q.N. 8(b)(or), 2064]**

9. In any triangle ABC, $b = \sqrt{3}$, $c = 1$ and $\angle A = 30°$, solve the triangle. **[Q.N. 8(b), (Or), 2065]**

10 If A=30°, B=45°, $a = 6\sqrt{2}$, solve the triangle ABC.
(Ans: $b = 12$, $c = 6(\sqrt{3} + 1)$, $C = 105°$) **[Q.N.8 (b), 2066]**

11. In any triangle ABC, $b = \sqrt{3}$, $C = 1$ and $A = 30°$, solve the triangle.
(Ans: $a = c = 1$, $b = \sqrt{3}$, $\angle A = \angle C = 30°$, $\angle B = 120°$) 4 **[Q.N.8 (b), 2067]**

12. If $a = 2$, $b = 1 + \sqrt{3}$, $C = 60°$, solve the triangle ABC. **[Q.N. 7(a)(Or), Set 'B' 2069]**
(Ans: $a = 2$, $b = 1 + \sqrt{3}$, $c = \sqrt{6}$, $A = 45°$, $B = 75°$, $C = 60°$)

Unit 5 – Sequence and Series, and Mathematical Induction

1. Using the principle of mathematical induction, prove that: $1+2+3+4+\ldots\ldots\ldots+n=\frac{n(n+1)}{2}$ [Q.N.2(b),2068]
2. Prove that: $1^2+2^2+3^2+4^2+\ldots\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$ [Q.N.12,2068]
3. Use the principle of mathematical induction: $2+4+6+\ldots\ldots\ldots+2n=n(n+1)$. [Q.N. 2(b), Set 'A' 2069]
4. Find the sum of n terms of the series $3.1^2+4.2^2+5.3^2+\ldots\ldots$ $\left(\text{Ans: } \frac{1}{12}n(n+1)(3n^2+11n+4)\right)$ [Q.N. 12, Set 'A' 2069]
5. Prove by the principle of mathematical induction. $2+4+6\ldots\ldots\ldots+2n=n(n+1)$. [Q.N. 2(b), Set 'B' 2069]
6. Find the n^{th} term and then the sum of the first n-terms of the series: $1^2.2+2^2.3+3^2.4+\ldots\ldots$ [Q.N. 12, Set 'B' 2069] $\left(\text{Ans: } n^3+n^2; \frac{n(n+1)(3n^2+5n+1)}{6}\right)$
7. Using principle of mathematical induction, prove that: $2+2^2+2^3+\ldots\ldots\ldots+2^n=2(2^n-1)$. [Q.N. 2(b), Supp. 2069]
8. The sum of an infinite number of terms in G.S. is 15, and the sum of their squars is 45; find the series. [Q.N. 12, Supp. 2069] $\left(\text{Ans: } 5+\frac{10}{3}+\frac{20}{9}+\ldots\right)$
9. Using principle of mathematical induction, prove that: $1+3+5+7+\ldots\ldots\ldots+(2n-1)=n^2$. [Q.N. 2(b), 2070 'C']
10. Show that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the following relations.
    a) $(G.M.)^2=A.M.\times H.M.$ b) $A.M.>G.M.>H.M.$ [Q.N. 12, 2070 'C']
11. Using principle of mathematical induction, prove that: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$. [Q.N. 2(b), 2070 'D']
12. Sum to infinity the following series. $1-5a+9a^2-13a^3+\ldots\ldots$ to ∞ $(-1<a<1)$. [Q.N. 12, 2070 'D'] $\left(\text{Ans: } \frac{1-3a}{(1+a)^2}\right)$

Unit 6 – Matrices and Determinants

1. Let $A=\begin{pmatrix}1&3\\2&4\end{pmatrix}$ and $B=\begin{pmatrix}5&7\\6&8\end{pmatrix}$ then prove $AB\neq BA$. [Q.N.3(a), 2056]
2. Evaluate $\begin{vmatrix}1+x&1&1\\1&1+y&1\\1&1&1+z\end{vmatrix}$ [Q.N.10(a), 2056]
   (Ans: $xyz+yz+zx+xy$)
3. Let $A=\begin{pmatrix}1&3\\2&4\end{pmatrix}$ and $B=\begin{pmatrix}3&2\\1&4\end{pmatrix}$ find the transpose of AB. [Q.N.3(a), 2057]
   $\left(\text{Ans: } \begin{pmatrix}6&10\\14&20\end{pmatrix}\right)$

4. Find the value of: $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ [Q.N.10(a), 2057]

(Ans: 0)

5. Let $A = \begin{pmatrix} 2 & 1 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix}$,

Find $(AB)^T$ [Q.N.3(a), 2058]

$\left(\text{Ans: } \begin{pmatrix} 5 & 21 \\ 12 & 14 \end{pmatrix}\right)$

6. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, show that $A^2 - 2A - 5I = 0$, [Q.N.10(a), 2058]

7. Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$, find $(AB)^T$. [Q.N.3(a), 2059]

$\left(\text{Ans: } \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix}\right)$

8. Evaluate $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ [Q.N.10(a), 2059]

(Ans: $(a-b)(b-c)(c-a)(a+b+c)$)

9. $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$, show that $(AB)^T = B^T A^T$. [Q.N.3(a), 2060]

10. Use properties of determinant to show that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$ [Q.N.10(a), 2060]

11. Construct a 3×3 matrix whose elements are $a_{ij} = 2i + j$. [Q.N.3(a), 2061]

$\left(\text{Ans: } \begin{pmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{pmatrix}\right)$

12. Without expanding show that :

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$ [Q.N.10(a), 2061]

13. If $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$, show that $(AB)^T = B^T A^T$ [Q.N.3(a), 2062]

14. Prove that : $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ [Q.N.10(a), 2062]

15. Define a triangular matrix. How do you distinguish between upper and lower triangular matrixes ? [Q.N.3(a), 2063]

16. Prove (without expanding).

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$ [Q.N.10(a), 2063]

17. If $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, find A^{-1} **[Q.N. 4(b), 2064]**

$\left(\textbf{Ans: } \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}\right)$

18. Prove (without expanding):

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$ **[Q.N. 10(a), 2064]**

19. Define symmetric and skew symmetric matrix with examples. **[Q.N. 3(a), 2065]**

20. Show that : $\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2$ **[Q.N. 10(a), 2065]**

21. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ then determine the marix A.

$\left(\textbf{Ans: } \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}\right)$ **[Q.N.3 (a), 2066]**

22. If a, b, c are non zero and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, then show that abc = 1.

[Q.N.10 (a), 2066]

23. Construct (3×3) matrix with the elements given by $a_{ij} = 2i + j$.

$\left(\textbf{Ans: } \begin{pmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{pmatrix}\right)$ 2 **[Q.N. 3 (a), 2067]**

24. Prove that : $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 2bxy + cy^2)$

4 **[Q.N.10 (a), 2067]**

25. Find the inverse of the matrix $A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$ **[Q.N.2(c),2068]**

$\left(\textbf{Ans: } \frac{1}{32}\begin{pmatrix} 2 & 3 \\ -6 & 7 \end{pmatrix}\right)$

26. Show that: $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$ **[Q.N.7(b),2068]**

27. If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$ find $(AB)^T$.

$\left(\text{Ans: } \begin{matrix} 13 & 4 \\ 22 & 3 \end{matrix}\right)$ **[Q.N. 2(c), Set 'A' 2069]**

28. Show that: $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c).$

[Q.N. 7(b), Set 'A' 2069]

29. For the given matrices $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$, so that: $(A + B)' = A' + B'$. **[Q.N. 2(c), Set 'B' 2069]**

30. Without expanding show that: $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0.$

[Q.N. 7(b), Set 'B' 2069]

31. If $A = \begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$, find Adj. A. **[Q.N. 2(c), Supp. 2069]**

$\left(\textbf{Ans: } \begin{pmatrix} 2 & -3 \\ -4 & 2 \end{pmatrix}\right)$

32. Prove that : $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c).$ **[Q.N. 7(b), Supp. 2069]**

33. If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find AA^T. **[Q.N. 2(c), 2070 'C']**

$\left(\textbf{Ans: } \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\right)$

34. Prove that: $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ **[Q.N. 7(b), 2070 'C']**

35. If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, fid $B^T A^T$. **[Q.N. 2(c), 2070 'D']**

$\left(\text{Ans: } \begin{pmatrix} 13 & 0 \\ 2 & 21 \end{pmatrix}\right)$

36. Prove that: $\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1 + \frac{a}{x} + \frac{b}{y} + \frac{c}{z}\right)$ **[Q.N. 7(b), 2070 'D']**

Unit 7 – System of Linear Equations

1. Solve by Cramer's rule: $-x + y = 9$, $x - 3y = 5$ **[Q.N.3(b), 2056]**
   (Ans: $x = -16$ and $y = -7$)
2. Solve by row equivalent matrix method: $x + z = 1$; $z + 2y = 2$; $5x - 9y = -3$ **[Q.N.10(b), 2056]**
   (Ans: $x = 3$, $y = 2$ and $z = -2$)
3. Solve by using inverse matrix method: $x - y = 2$; $2x + 3y = 9$ **[Q.N.10(b) (Or), 2056]**
   (Ans: $x = 3$, $y = 1$)
4. Solve by Cramer's rule : $2x - y = 5$; $x - 2y = 1$. **[Q.N.3(b), 2057]**
   (Ans: $x = 3$, $y = 1$)
5. Solve by using row equivalent matrix method: **[Q.N.10(b), 2057]**
   $x - 2y + 2z = 0$; $x - 2y + 3z = -1$; $2x - 2y + z = -3$
   (Ans: $x = -4$, $y = -3$, $z = -1$)

6. Solve by inverse matrix method: $-2x + 4y = 3$
$3x - 7y = 1$ [Q.N.10(b) (Or), 2057]

$\left(\textbf{Ans: } x = \frac{-25}{2}, y = \frac{-11}{2}\right)$

7. Solve by Cramer's rule,
$x - 2y = -7$
$3x + 7y = 5$ [Q.N.3(b), 2058]
(Ans: x = –3, y = 2)

8. Solve, by row equivalent matrix method,
$x + y + z = 1$
$x + 2y + 2z = 4$
$x + 3y + 7z = 13$ [Q.N.10(b), 2058]
(Ans: x = 1, y = –3, z = 3)

9. Solve by inverse matrix method,
$2x + 5y = 7, \quad 5x + 2y = -3$ [Q.N.10(b) (Or), 2058]

$\left(\textbf{Ans: } x = \frac{-29}{21}, y = \frac{41}{21}\right)$

10. Solve by Cramer's rule $2x - y = 5; \quad x - 2y = 1.$ [Q.N.3(b), 2059]
(Ans: x = 3, y = 1)

11. Solve by row equivalent matrix method $x + z = 1; \quad z + 2y = 2; \quad 5x - 9y = -3.$
[Q.N.10(b), 2059]
(Ans: x = 3, y = 2, z = –2)

12. Solve by using inverse matrix method: $\left.\begin{matrix} 2x + 4y = 7 \\ 8x - 6y = -5 \end{matrix}\right\}$ [Q.N.10(b) (Or), 2059]

$\left(\textbf{Ans: } x = \frac{1}{2} \text{ and } y = \frac{3}{2}\right)$

13. Give reason why simultaneous equation $x + 2y = 5$ and $3x + 6y = 12$ are not solvable by Cramer's rule. [Q.N.3(b), 2060]

14. Solve, by row equivalent matrix method :
$x - 2y - 3z = -1, 2x + y + z = 6, x + 3y - 2z = 13.$ [Q.N.10(b), 2060]
(Ans: x = 2, y = 3 and z = –1)

15. Solve by matrix inversion method $3x + 5y = 7, 5x + 9y = 7.$ [Q.N.10(b) (Or), 2060]
(Ans: x = 14 and y = – 7)

16. Solve by matrix inversion method, $x + y = 2$ and $x - y = 0.$ [Q.N.3(b), 2061]
(Ans: x = 1 and y = 1)

17. Solve by Cramer's rule :
$x + 2y + 3z = 6, 2x + 4y + z = 7$ and $3x + 2y + 9z = 14.$ [Q.N.10(b), 2061]
(Ans: x =1, y = 1 and z = 1)

18. Solve by Row–equivalent matrix method :
$x - y - z = -2, x + 4z = 4$ and $y - 2z = 1.$ [Q.N.10(b) (Or), 2061]

$\left(\textbf{Ans: } x = \frac{8}{7}, y = \frac{17}{7}, z = \frac{5}{7}\right)$

19. Solve: $x = 2y + 3$ and $3x - 5y = 8$ by inverse matrix method. [Q.N.3(b), 2062]
(Ans: x = 1 and y = –1)

20. Solve by row- equivalent method:
$9y - 5x = 3,$
$x + z = 1$ and
$z + 2y = 2$ [Q.N.10(b), 2062]
(Ans: x = 3, y = 2 and z = –2)

21. Solve the equations by Cramer's rule :
$9y - 5x = 3,$
$x + z = 1$ and
$z + 2y = 2$ [Q.N.10(b) (Or), 2062]
(Ans: x = 3, y = 2 and z = −2)

22. Solve by Inverse matrix method :
$3x + 2y = 5$
$7x + 5y = 12$ [Q.N.4(b), 2063]
(Ans: x = 1, y = 1)

23. Solve by Cramer's rule **or** by row equivalent matrix method.
$x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$ [Q.N.10(b), 2063]
(Ans: x = 1, y = 2, z = 3)

24. Solve by row-equivalent method:
$3x - 2y = 8$
$5x + 3y = 7$ [Q.N. 3(a), 2064]
(Ans: 2, −1)

25. Solve the following system of equations by inverse matrix method **or** Cramer's rule:
$2x - y + 3z = 9, x + y + z = 6, x - y + z = 2.$ [Q.N. 10(b), 2064]
(Ans: 1, 2, 3)

26. Solve by Cramer's rule :
$3x + 2y = 8,$
$4x + y = 9$ [Q.N. 3(b), 2065]
(Ans: x = 2, y = 1)

27. Solve by matrix method :
$x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$ [Q.N. 10(b), 2065]
(Ans: x = 1, y = 2, z = 3)

28. Solve by cramer's rule : $-x + y = 9, x - 3y = 5$ [Q.N.3 (b), 2066]
(Ans: (−16, −7))

29. Solve by row equivalent or inverse matrix method:
$x + z = 1,$
$z + 2y = 2,$
$5x - 9y = -3$
(Ans: 3, 2, − 2) [Q.N.10 (b), 2066]

30. Solve by matrix inversion method : $x + y = 3$ and $x - y = 1$. 2 [Q.N. 3 (b), 2067]
(Ans: 2, 1)

31. Solve by Cramer's rule or Row-equivalent method :
$x + y + z = 9, 2x + 5y + 7z = 52, 2x + y - z = 0$ 4 [Q.N.10 (b), 2067]
(Ans: 1, 3, 5)

32. Using Cramer's rule, solve the system of equations:
$2x + 5y = 17$
$5x - 2y = -1$ [Q.N.3(a),2068]
(Ans: x = 1, y = 3)

33. Applying row equivalent matrix method or inverse matrix method, solve the following system of equations:
$x + y + z = 1$
$x + 2y + 3z = 4$
$x + 3y + 7z = 13$ [Q.N.8(a),2068]
(Ans: x = 1, y = −3, z = 3)

34. Using Cramer's rule, solve the following equations:

$x - 2y = -7$

$3x + 7y = 5$

[Ans: –3, 2] **[Q.N. 3(a), Set 'A' 2069]**

35. Using row equivalent matrix method or inverse matrix method, solve the following equations.

$x - 2y - z = -7$

$2x + y + z = 0$

$3x - 5y + 8z = 13$

(Ans: –2, 1, 3) **[Q.N. 8(a), Set 'A' 2069]**

36. Solve by Cramer's rule: $3x + 2y + 9 = 0$, $2x - 3y + 6 = 0$ **[Q.N. 3(a), Set 'B' 2069]**

(Ans: –3, 0)

37. Using raw equivalent or inverse matrix method, solve the following system of equations.

$x - y = 0$, $2x - y + 4z = 18$, $-3x + z + 2 = 0$

(Ans: 2, 2, 4) **[Q.N. 8(a), Set 'B' 2069]**

38. Using Cramer's rule, solve the following equations.

$2x - 5y = 24$

$2x + 3y = 12$ **[Q.N. 3(a), Supp. 2069]**

(Ans: 8.25, – 1.5)

39. Using row equivalent matrix method or inverse matrix method, solve the following system.

$3x + 5y = 2$

$2x - 3z = -7$

$4y + 2z = 2$ **[Q.N. 8(a), Supp. 2069]**

(Ans: 4, – 2, 5)

40. Using Cramer's rule, solve the following equations:

$3x - 2y = 8$, $5x + 3y = 7$. **[Q.N. 3(a), 2070 'C']**

(Ans: x = 2, y = – 1)

41. Using row equivalent matrix method **or** inverse matrix method, solve the following equations.

$x + 4y + z = 18$, $3x + 3y - 2z = 2$, $-4y + z = -7$ **[Q.N. 8(a), 2070 'C']**

(Ans: x = 1, y = 3, z = 5)

42. Applying Cramer's rule, solve the following equations:

$3x + \frac{4}{y} = 10$, $-2x + \frac{3}{y} = -1$. **[Q.N. 3(a), 2070 'D']**

(Ans: 2, 1)

43. Using row equivalent matrix method **or** inverse matrix method, solve the following equations.

$9y - 5x = 3$, $x + z = 1$, $z + 2y = 2$ **[Q.N. 8(a), 2070 'D']**

(Ans: 3, 2, – 2)

Unit 8 – Complex Number

1. If $z = 3 + 4i$ and $w = 2 + i$, find $|zw|$ and $\left|\frac{z}{w}\right|$. **[Q.N.4(a), 2056]**

(Ans: $5\sqrt{5}$ and $\sqrt{5}$)

2. State De- Moivre's theorem. Use it to find the cube roots of unity. **[Q.N.11(a), 2056]**

$\left(\textbf{Ans: } 1, \frac{-1 + i\sqrt{3}}{2} \textbf{ and } \frac{-1 - i\sqrt{3}}{2}\right)$

3. If $Z_1 = (3, 2)$; $Z_2 = (5, 3)$, compute $Z_1 Z_2$ and $\overline{Z_1 + Z_2}$ **[Q.N.4(b), 2057]**

(Ans: $Z_1Z_2 = (9, 19)$ and $\overline{Z_1 + Z_2} = (8, -5)$)

4. State De-Moivre's Theorem. Use it to find the values of $(1+i)^{20}$ **[Q.N.11(b), 2057]**

(Ans: -2^{10})

5. Express the complex number $(2, 2\sqrt{3})$ in the polar form. **[Q.N.4(a), 2058]**
(Ans: 4 [cos60° + i sin 60°])

6. State De-Moivre's theorem. Use it to find the cube roots of 1. **[Q.N.11(a), 2058]**
$\left(\textbf{Ans: } 1, \frac{-1+i\sqrt{3}}{2} \& \frac{-1-i\sqrt{3}}{2}\right)$

7. Express in the polar form for z = 2 + 2i. **[Q.N.4(a), 2059]**
(Ans: $2\sqrt{2}$ (cos45° + *i* sin45°))

8. Find the square roots of z = 7 − 24i. **[Q.N.11(a), 2059]**
(Ans: ± (4 − 3*i*))

9. Express the complex number, $-\sqrt{2} + i\sqrt{2}$ in polar form. **[Q.N.4(a), 2060]**
(Ans: 2 (cos 135° + *i* sin 135°))

10. Using De Moiver's theorem, find the fourth roots of unity. **[Q.N.11(a), 2060]**
(Ans: 1, *i*, −1, −*i*)

11. Find the conjugate of the complex number $\frac{3+4i}{3-4i}$ **[Q.N.4(a), 2061]**
$\left(\textbf{Ans: } Z = \overline{Z} = \frac{-7}{25} - \frac{24}{25}i\right)$

12. State De Moivre's theorem hence solve $z^6 = 1$. **[Q.N.11(a), 2061]**
$\left(\textbf{Ans: } \pm 1, \pm\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right), \pm\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right)$

13. Simplify : $[3 (\text{Cos}\,\frac{\pi}{4} + i\,\text{Sin}\,\frac{\pi}{4})]^{16}$ **[Q.N.4(a), 2062]**
(Ans: 3^{16})

14. Find the cube roots of unity. Write their properties. **[Q.N.11(a), 2062]**
$\left(\textbf{Ans: } 1, \frac{-1+i\sqrt{3}}{2} \text{ and } \frac{-1-i\sqrt{3}}{2}\right)$

15. Find the square roots of 7 + 24*i* **[Q.N.3(b), 2063]**
(Ans: ± (4 + 3i))

16. Find the cube roots of unity and discuss their properties. **[Q.N.12(b), 2063]**
$\left(\textbf{Ans: } 1, \frac{-1+i\sqrt{3}}{2} \text{ and } \frac{-1-i\sqrt{3}}{2}\right)$

17. If 1, w, w^2 be the cube roots of unity, prove that $(1 + w^2)^3 - (1 + w)^3 = 0$
[Q.N. 3(b), 2064]

18. Solve : $z^6 = 1$. **[Q.N. 12(a), 2064]**
$\left(\textbf{Ans: } \pm 1, \pm\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right), \pm\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right)$

19. If α, β are the complex cube roots of unity then show that :
$$\alpha^4 + \beta^4 + \frac{1}{\alpha\beta} = 0$$ **[Q.N. 4(a), 2065]**
(Ans: 0)

20. Find the square roots of (−7 + 24i). **[Q.N. 11(a), 2065]**
(Ans: ± (3 + 4i))

21. Prove that $(2+\omega)(2+\omega^2)(2-\omega^2)(2-\omega^4) = 21$ **[Q.N.4 (a), 2066]**