22. If $\sqrt{x+iy} = a + ib$, prove that $\sqrt{x-iy} = a - ib$ [Q.N.11 (a), 2066]

23. Express the complex number $\frac{i}{1-i}$ in the polar form.
$\left(\text{Ans: } \frac{1}{\sqrt{2}}(\cos 135° + i \sin 135°)\right)$ [Q.N. 4 (a), 2067]

24. Find the square root of $\frac{5+12i}{3-4i}$ [Q.N.11 (a), 2067]
$\left(\text{Ans: } \pm \frac{3+2i}{2-i}\right)$

25. Find the cube roots of unity. [Q.N.3(b),2068]
$\left(\text{Ans: } 1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right)$

26. State De-Moivre's theorem. Using De Moivre's theorem, find the square roots of $-2-2\sqrt{3}\,i$. [Q.N.14,2068]
(Ans: ± $(1-i\sqrt{3})$)

27. If w be a complex cube root of unity, find the value of:
$(1-w+w^2)^4\,(1+w-w^2)^4$
(Ans: 256) [Q.N. 3(b), Set 'A' 2069]

28. If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, prove that:
$z_1 z_2 = r_1 r_2\{\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\}$ and
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\{\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\}$ [Q.N. 14, Set 'A' 2069]

29. Define complex number. Express a complex number into polarform. State De-Moivre's theorem. Using De'Moivre's theorem, find the cube roots of unity.
$\left(\text{Ans: } 1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right)$ [Q.N. 14(Or), Set 'A' 2069]

30. Express $\sqrt{3}$ +i in polar form. [Q.N. 3(b), Set 'B' 2069]
(Ans: 2(cos 30º + i sin 30º)

31. State De Moivre's theorem for any positive index n. Using De Moivre's theorem find the square roots of $4 + 4\sqrt{3}\,i$. [Q.N. 14, Set 'B' 2069]
(Ans: (± $\sqrt{6} + i\sqrt{2}$)

32. If $x = a + b$, $y = a\omega + b\omega^2$ and $z = a\omega^2 + b\omega$, show that:
$x + y + z = 0$. [Q.N. 3(b), Supp. 2069]

33. State De'Moiver's theorem and use it to solve the equation $Z^6=1$.
[Q.N. 14, Supp. 2069]
$\left(\text{Ans: } \pm 1, \frac{1}{2}(1 \pm i\sqrt{3}), \frac{1}{2}(-1 \pm i\sqrt{3})\right)$

34. If $\alpha = \frac{1}{2}(-1+\sqrt{-3})$, $\beta = \frac{1}{2}(-1-\sqrt{-3})$,
Show that : $\alpha^4 + \alpha^2\beta^2 + \beta^4 = 0$. [Q.N. 3(b), 2070 'C']

35. Find the square root of the complex number −5 + 12i. [Q.N. 14, 2070 'C']
(Ans: ± (2 + 3i))

36. Find the values of x and y if $(x + 2) + yi = (3 + i)(1 - 2i)$.
(Ans: 3, − 5) [Q.N. 3(b), 2070 'D']

37. Define absolute value of a complex number. If Z and W are two complex numbers, prove that: $|z + w| \le |z| + |w|$ [Q.N. 14, 2070 'D']

38. Find the cube roots of unity. Also, establish the properties of cube roots of unity. [Q.N. 14(Or), 2070 'D']

Unit 9 – Polynomial Equations

1. For what value of p will the equation $5x^2 - px + 45 = 0$ have equal roots ?
(Ans: ± 30) [Q.N.4(b), 2056]

2. If the roots of the equation $lx^2 + nx + n = 0$ be in the ratio p:q, find the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}$ [Q.N.11(b), 2056]
$\left(\text{Ans: } -\sqrt{\frac{n}{\ell}}\right)$

3. Form the quadratic equation whose one root is 3 + 4i. [Q.N.4(a), 2057]
(Ans: $x^2 - 6x + 25 = 0$)

4. Find the condition for two given quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ may have one root common and both roots common. [Q.N.11(a), 2057]
$\left(\text{Ans: } (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1)^2 \text{ and } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$

5. Is (x – 2) a factor of $x^3 + 3x^2 - 5x + 2$? Justify your answer. [Q.N.4(b), 2058]
(Ans: (x – 2) is not a factor of $x^3 + 3x^2 - 5x + 2$)

6. If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that, $b^3 + a^3c + ac^2 = 3abc$ [Q.N.11(b), 2058]

7. Is (x – 2) a factor of $x^3 + 3x^2 - 5x + 2$? If not, find the remainder. [Q.N.4(b), 2059]
(Ans: x – 2 is not a factor of $x^3 + 3x^2 - 5x + 2$)

8. Show that the roots of the equation $(a^2 - bc)x^2 + 2(b^2 - ca)x + (c^2 - ab) = 0$ will be equal, if either b = 0 or $a^3 + b^3 + c^3 - 3abc = 0$. [Q.N.11(b), 2059]

9. If the roots of the quadratic equation are p+q and p–q, find the quadratic equation. [Q.N.4(b), 2060]
(Ans: $x^2 - 2px + p^2 - q^2 = 0$)

10. If the roots of $\ell^2x^2 + mx + n = 0$ be in the ratio 3 : 4, show that $12m^2 = 49\ell n$. [Q.N.11(b), 2060]

11. Apply remainder theorem to find the remainder when $x^3 - 2x^2 + 5x - 10$ is divided by x + 2 [Q.N.4(b), 2061]
(Ans: –36)

12. If one root of the equation $\ell x^2 + mx + n = 0$ be four times the other, show that $4m^2 = 25\ell n$. [Q.N.11(b), 2061]

13. State the factor theorem & test whether $x + 1$ is the factor of $2x^3 - 4x^2 + 5x - 1$ or not ? [Q.N.4(b), 2062]
(Ans: $x + 1$ is not a factor of $f(x) = 2x^3 - 4x^2 + 5x - 1$)

14. The quadratic equation : $ax^2 + bx + c = 0$ can not have more than two roots. Prove it. [Q.N.11(b), 2062]

15. When $2x^3 + 3x^2 - Kx + 4$ divided by $x - 2$, the remainder is 2K, find the value of K. [Q.N.4(c), 2063]
(Ans: k = 8)

16. Under what conditions will quadratic equation $ax^2 + bx + c = 0$ has
i. one root the reciprocal of the other.
ii. roots equal in magnitude but opposite in sign.
(Ans: (i) c = a (ii) b = 0) [Q.N.11(b), 2063]

17. Find out which of the following are factors of $2x^3 - 3x^2 - 9x + 10$:
(i) $x - 1$ (ii) $x + 1$ (iii) $x - 2$ (iv) $x + 2$ **[Q.N. 4(c), 2064]**
(Ans: (i), (iv))

18. Under what conditions are the roots of the quadratic equation $ax^2 + bx + c = 0$.
(i) real and unequal (ii) imaginary **[Q.N. 11(b), 2064]**
(Ans: (i) $b^2 - 4ac > 0$ (ii) $b^2 - 4ac < 0$)

19. For what value of k, $x + 3$ is a factor of $3x^2 + kx + 6$? **[Q.N. 4(b), 2065]**
(Ans: 11)

20. Find the equation whose roots are reciprocal to the roots of $x^2 - x + 1 = 0$
(Ans: $(x^2 - x + 1)$) **[Q.N. 11(b), 2065]**

21. Find the remainder when x^3+6x^2-x-30 is divided by $x+1$.
(Ans: – 24) **[Q.N.4 (b), 2066]**

22. If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. **[Q.N.11 (b), 2066]**

23. For what value of k the polynomial $2x^3 - 3x^2 - kx + 4$ divided by $x - 2$ gives remainder 2k ? 2 **[Q.N.4 (b), 2067]**
(Ans: 2)

24. If one root of the equation $ax^2 + bx + c = 0$ is triple of the other, show that $3b^2 = 16ac$. 4 **[Q.N.11 (b), 2067]**

25. If the roots of the equation $(a^2 + b^2)\, x^2 - 2(ac + bd)x + (c^2, + d^2) = 0$ are equal, then show that: $\frac{a}{b} = \frac{c}{d}$ **[Q.N.8(b),2068]**

26. Form a quadratic equation whose roots are – 5 and 4. **[Q.N.3(c),2068]**
(Ans: $x^2 + x - 20 = 0$)

27. For what values of p will the equation $5x^2 - px + 45 = 0$ have equal roots.
(Ans: ± 30) **[Q.N. 3(c), Set 'A' 2069]**

28. Prove that a quadratic equation cannot have more that two roots.
[Q.N. 8(b), Set 'A' 2069]

29. If one root of the equation $ax^2 + bx + c = 0$ be twice the other show that: $2b^2 = 9ac$. **[Q.N. 3(c), Set 'B' 2069]**

30. From the equation whose roots are the reciprocals of the roots of $ax^2 + bx + c = 0$. **[Q.N. 8(b), Set 'B' 2069]**
(Ans: $cx^2 + bx + a = 0$)

31. Find the value of k so that the equation $3x^2 + 7x + 6 - k = 0$ has one root equal to zero. **[Q.N. 3(c), Supp. 2069]**
(Ans: 6)

32. If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that: $p = q$ or $p+q+1= 0$. **[Q.N. 8(b), Supp. 2069]**

33. If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k.
(Ans: 1, 4) **[Q.N. 3(c), 2070 'C']**

34. Find the condition under which the two quadratic equations $ax^2 + bx + c = 0$ and $a^1x^2 + b^1x + c^1 = 0$ may have one root common.
(Ans: $(ab' - a'b)(bc' - b'c) = (a'c - ac')^2$) **[Q.N. 8(b), 2070 'C']**

35. Find the quadratic equation whose one root is $2 + \sqrt{3}$.
(Ans: $x^2 - 4x + 1 = 0$) **[Q.N. 3(c), 2070 'D']**

36. If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3+a^2c + ac^2 = 3abc$. **[Q.N. 8(b), 2070 'D']**

Unit 10 – Co-ordinate Geometry

10.1 Straight Line

1. Find the acute angle between the lines $x - 3y - 6=0$ and $y=2x+5$.
   (Ans: 45°) [Q.N.2(b), 2056]

2. Find the angles between two lines given by $y = m_1x + c_1$ and $y = m_2x + c_2$. Also state the condition for them to be perpendicular and parallel.
   $\left(\text{Ans: } \phi = \tan^{-1}\left(\pm\dfrac{m_1 - m_2}{1 + m_1m_2}\right)\right)$ [Q.N.9(a), 2056]

3. What are the standard forms of equation of a straight line ? Find the slope of the line $\frac{x}{a} - \frac{y}{b} = 1$. [Q.N.2(b), 2057]
   $\left(\text{Ans: Slope}(m) = \dfrac{b}{a}\right)$

4. Find the length of the perpendicular from the point (x_1, y_1) on a straight line $x \cos\alpha + y \sin\alpha = p$. [Q.N.9(a), 2057]
   (Ans: $\pm (x_1\cos\alpha + y_1\sin\alpha - P)$)

5. Write the conditions for which the straight lines given by $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ will be parallel and perpendicular [Q.N.2(b), 2058]
   $\left(\text{Ans: } \phi = \tan^{-1}\left(\pm\dfrac{m_1 - m_2}{1 + m_1m_2}\right);\ m_1m_2 = -1;\ m_1 = m_2\right)$

6. Find the length of the perpendicular from the point (h, k) on a straight line $x \cos\alpha + y \sin\alpha = p$ [Q.N.9(a), 2058]
   (Ans: $\pm (h\cos\alpha + k\sin\alpha - p)$)

7. Find the equation of the line through (5, 4) and perpendicular to the line $4x - 3y = 10$. [Q.N.2(b), 2059]
   (Ans : $3x + 4y - 31 = 0$)

8. Find the length of the perpendicular from the point (x_1, y_1) on a straight line $x \cos\alpha + y \sin\alpha = p$. [Q.N.9(a), 2059]
   (Ans: $\pm (x_1\cos\alpha + y_1\sin\alpha - p)$)

9. Find the equation of the straight line whose slope is $\frac{1}{3}$ and passes through the intersection of lines $y = x$ and $y = -x$. [Q.N.2(b), 2060]
   (Ans: $x - 3y = 0$)

10. Find the equation of the line through the point that divides the join of the points (–3, –4) and (7, 1) in the ratio 3 : 2 and is perpendicular to the join.
    (Ans: $-2x - y + 5 = 0$) [Q.N.9(a), 2060]

11. Find the straight lines which have slope – 1 and form a triangle of area 8 square units with coordinate axes. [Q.N.2(b), 2061]
    (Ans: $x + y - 4 = 0$)

12. Find the equation to the straight line which passes through the intersection of the straight lines $3x - 4y + 1 = 0$ and $5x + y = 1$, and cuts off equal intercepts from the axes. [Q.N.9(a), 2061]
    (Ans: $23x + 23y = 11$)

13. Find the distance between the parallel lines, $y = 2x + 4$ and $6x - 3y = 5$. [Q.N.2(b), 2062]
    $\left(\text{Ans: } \dfrac{17}{3\sqrt{5}} \text{ unit}\right)$

14. Find the equation of the locus of a point P which is equidistant from $3x - 4y + 2 = 0$ and the origin. **[Q.N.9(a), 2062]**
(Ans: $16x^2 + 9y^2 + 24xy - 12x + 16y - 4 = 0$)

15. Find the intercepts on the axes made by the line $2x + 3y = 5$ **[Q.N.2(b), 2063]**
$$\left(\textbf{Ans: x-intercept} = \frac{5}{2} \textbf{ and y-intercept} = \frac{5}{3}\right)$$

16. Prove that the equation of the straight line which passes through the point $(a\cos^3\theta, a\sin^3\theta)$ and is perpendicular to the straight line $x \operatorname{Sec}\theta + y \operatorname{Cosec}\theta = a$ is $x \operatorname{Cos}\theta - y \operatorname{Sin}\theta = a \operatorname{Cos}2\theta$. **[Q.N.9(a), 2063]**

17. Find the equations of the bisectors of the angles between the straight lines $3x - 4y + 3 = 0$ and $12x - 5y - 1 = 0$. **[Q.N.9(a) Or, 2063]**
(Ans: $21x + 27y - 44 = 0$ and $99x - 77y + 34 = 0$)

18. If p be the perpendicular distance of the origin from a line whose intercepts on the axes are a and b, prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ **[Q.N. 2(b), 2064]**

19. Find the equations of the straight lines which passes through the point (2, 3) and are inclined at 45° to the straight line $x + 3y + 4 = 0$. **[Q.N. 9(b), 2064]**
(Ans: ($x - 2y + 4 = 0$, $2x + y - 7 = 0$))

20. Find the equation of the line through the intersection of the lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$, and cutting off equal intercepts from the axes.
(Ans: $23x + 23y = 11$) **[Q.N. 2(b), 2065]**

21. The origin is a corner of a square and two of its sides are given by $2x + y = 0$ and $2x + y = 3$. Find the equations of the other two sides. **[Q.N. 9(a), 2065]**
(Ans: $2y - x = 0$ & $x - 2y \pm 3 = 0$)

22. Examine whether the points (0, 11), (2, 3) and (3, –1) are collinear or not.
(Ans: collinear) **[Q.N.2 (a), 2066]**

23. Determine the value of m for which the straight lines $y = x + 1$, $y = 2(x+1)$ and $y = mx + 3$ are concurrent.
(Ans: 3) **[Q.N.9 (a), 2066]**

24. Find the value of k so that the line whose equation is $x + y = k$ will form a triangle with the coordinate axes whose area is 32 sq. units.
(Ans: ± 8) 2 **[Q.N. 2 (b), 2067]**

25. Find the equation to the straight line which makes equal intercepts on the axes and passes through the point of intersection of the lines $2x - 3y + 1 = 0$ and $x + 2y - 2 = 0$.
(Ans: $7x + 7y = 9$) 4 **[Q.N.9 (a), 2067]**

26. Find the equation of the line parallel to the line $5x + 4y = 9$ and making an intercept –5 on the x-axis. **[Q.N.4(a),2068]**
(Ans: $5x + 4y + 25 = 0$

27. Find the angle between two straight lines whose equations are $y = m_1 x + c_1$ and $y = m_2 x + c_2$.
Also find the conditions under which the two straight lines will be
(i) parallel
(ii) perpendicular. **[Q.N.13,2068]**
$$\textbf{(Ans: } \theta = \tan{-1}\left(\pm \frac{m_1 - m_2}{1 + m_1 m_2}\right); \text{(i) } m_1 = m_2, \text{(ii) } m_1.m_2 \ -1)$$

28. Find the equation of a line through (5,4) and perpendicular to the line
$4x - 3y = 10$.
(Ans: $3x + 4y - 31 = 0$) **[Q.N. 4(a), Set 'A' 2069]**

29. If p and P^1 be the lengths of the perpendiculars from origin upon the straight lines whose equations are $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ prove that: $4p^2 + p^{1^2} = a^2$. **[Q.N. 13, Set 'A' 2069]**

30. Find the equation to the straight line that has y-intercepts 3 and is parallel to the straight line $8x - 4y + 9 = 0$.
(Ans: $2x - y + 3 = 0$) **[Q.N. 4(a), Set 'B' 2069]**

31. Prove that the perpendicular from the origin upon the straight line joining the points $(c\cos\alpha, c\sin\alpha)$ and $(c\cos\beta, c\sin\beta)$ bisects the distance between them. **[Q.N. 13, Set 'B' 2069]**

32. If p is the length of the perpendicular dropped from the origin of the line $\frac{x}{a} + \frac{y}{b} = 1$, prove that: $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$. **[Q.N. 4(a), Supp. 2069]**

33. Find the equations of the lines through the point (3, 2) and making angle 45º with the line $x - 2y = 3$. **[Q.N. 13, Supp. 2069]**
(Ans: $x + 3y - 9 = 0$, $3x - y - 7 = 0$)

34. Find the distance between the two parallel lines.
$3x + 5y = 11$ and $3x + 5y = -23$ **[Q.N. 4(a), 2070 'C']**
(Ans: $\sqrt{34}$)

35. Find the length of the perpendicular drown from the point (x^1, y^1) on the line whose equation is $Ax + By + c = 0$. **[Q.N. 13, 2070 'C']**
$\left(\text{Ans: } \left|\frac{Ax' + By' + C}{\sqrt{A^2 + B^2}}\right|\right)$

36. Find the equation of the line passing through the middle point of the line segment connecting (2, –4) and (2, 4) and parallel to the line $3x - 2y = 4$.
(Ans: $3x - 4y - 6 = 0$) **[Q.N. 4(a), 2070 'D']**

37. Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$ and prove that the bisectors are at right angles to each other. **[Q.N. 13, 2070 'D']**
(Ans: $4x + 7y + 11 = 0$, $7x - 4y + 3 = 0$)

10.2 Pair of Straight Lines

1. Write the condition for which the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a line pair. **[Q.N.2(c), 2056]**
(Ans: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$)

2. Prove that the straight lines joining the origin to the points of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$, and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. **[Q.N.9(b), 2056]**

3. Determine the lines represented by the equation $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$. **[Q.N.2(c), 2057]**
(Ans: $x + y - 5 = 0$ and $x + y + 3 = 0$)

4. If the pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2q\,xy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove $pq = -1$. **[Q.N.9(b), 2057]**

5. Find the angle between the line pair given by: $x^2 - 2xy\cot\theta - y^2 = 0$
(Ans: $\alpha = 90°$) **[Q.N.2(c), 2058]**

6. Prove that the pair of straight lines joining the origin to the points of intersection of the line $y = mx + c$ and the curve $x^2 + y^2 = a^2$ are at right angles of $2c^2 = a^2(1 + m^2)$ **[Q.N.9(b), 2058]**

7. Find the angle between the line pair $2x^2 + 7xy + 3y^2 = 0$. **[Q.N.2(c), 2059]**
(Ans: $\theta = 45°$ or $135°$)

8. Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a}+\frac{y}{b}=1$ and the curve $x^2+y^2=c$ are at right angles if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{2}{c^2}$.

[Q.N.9(b), 2059]

9. Verify whether the second degree equation $x^2+6xy+9y^2+4x+12y-5=0$ represents a pair of straight lines or not.

[Q.N.2(c), 2060]

10. Show that the lines joining the points of intersection of the line $x+y=1$ with the curve $4x^2+4y^2+4x-2y-5=0$ with the origin are at right angles to each other.

[Q.N.9(b), 2060]

11. Find the angle between the pair of lines $x^2+6xy+9y^2+4x+12y-5=0$.

(Ans: $\theta = 0°$)

[Q.N.2(c), 2061]

12. For what values of C, the lines which join the origin to the point of intersection of the line $x-y+c=0$ and the curve $x^2+y^2+4x-6y-36=0$ may be at right angles.

[Q.N.9(b), 2061]

(Ans: either c = 9, or c = −4)

13. Find the value of K so that $2x^2+7xy+3y^2-4x-7y+K=0$ may represent a pair of lines.

[Q.N.2(c), 2062]

(Ans: k = 2)

14. Determine the two straight lines represented by :

$6x^2-xy-12y^2-8x+29y-14=0$

[Q.N.9(b), 2062]

(Ans: 2x−3y+2=0, 3x+4y−7=0)

15. Find the angle between the lines represented by $2x^2+7xy+3y^2=0$

[Q.N.2(c), 2063]

(Ans: 45° or 135°)

16. Find the equations of the two lines represented by $x^2+6xy+9y^2+4x+12y-5=0$. Prove that the two lines are parallel.

[Q.N.9(b), 2063]

(Ans: x + 3y = 1 and x + 3y = −5)

17. Find the equations of the two lines represented by the equation $2x^2+3xy+y^2+5x+2y-3=0$

[Q.N. 2(c), 2064]

(Ans: x + y + 3 = 0, 2x + y − 1 = 0)

18. Find the single equation of the lines through the origin and perpendicular to the lines represented by the equation $ax^2+2hxy+by^2=0$.

[Q.N.9(a), 2064]

$\left(\text{Ans: } \tan^{-1}\left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)\right)$

19. Show that the equation $kx^2+(k^2-1)xy-ky^2=0$ represents a pair of perpendicular lines for all values of k.

[Q.N. 2(c), 2065]

20. Show that pair of lines $x^2(\tan^2\theta+\cos^2\theta)-2xy\tan\theta+y^2\sin^2\theta=0$ make with the axis of x angle such that the difference of their tangent is 2. **[Q.N. 9(b), 2065]**

21. For what vaue of K, the equation $2x^2+7xy+3y^2-4x-7y+K=0$ represents a line pair?

(Ans: 2)

[Q.N.2 (c), 2066]

22. Find the equation of the lines which are right angles to the lines represented by $ax^2+2hxy+by^2=0$.

[Q.N.9 (b), 2066]

(Ans: $bx^2-2hxy+ay^2=0$)

23. Find the angle between the lines given by $x^2-2xy\cot\theta-y^2=0$.

(Ans: 90°)

2 **[Q.N. 2 (c), 2067]**

24. Find the condition so that the straight lines joining the origin to the points of intersection of the line kx + hy = 2hk with the circle $(x - h)^2 + (y - k)^2 = c^2$ are at right angle.
(Ans: $h^2 + k^2)(h^2 + k^2 - c^2 - 4hk) + 8h^2k^2) = 0$ 4 [Q.N.9 (b), 2067]

25. Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if: $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$
[Q.N.13(Or),2068]

26. Show that the homogeneous equation of degree two always represents a pair of straight line passing through the origin. Also, find the angle between them.
[Q.N. 13(Or), Set 'A' 2069]

27. Prove that the bisectors of the angles between the pair of straight lines $ax^2 - 2hxy + by^2 = 0$ is given by $\frac{x^2 - y^2}{xy} = \frac{a - b}{h}$ [Q.N. 13(Or), Set 'B' 2069]

28. Find the angle between the two lines represented by $ax^2 + 2hxy + by^2 = 0$.
Find the condition under which the lines will be
i) perpendicular to each other.
ii) coincident.
What condition is to be satisfied for two lines to be real and distinct?
[Q.N. 13(Or), Supp. 2069]
(Ans: $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$, (i) $a + b = 0$, (ii) $h^2 - ab = 0$, $h^2 - ab > 0$)

29. Find the equation to the pair of lines joining the origin to the inter section of the straight line $y = mx + c$ and the curve $x^2 + y^2 = a^2$. Prove that they are at right angles if $2c^2 = a^2(1 + m^2)$. [Q.N. 13(Or), 2070 'C']
(Ans: $(c^2 - a^2m^2)x^2 + 2a^2m\,xy + (c^2 - a^2)y^2 = 0$)

30. Find the condition under which the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of lines. [Q.N. 13(Or), 2070 'D']
(Ans:: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$)

Unit 11 – Circle

1. Find the equation of the circle with center at (4, –1) and passing through the origin. [Q.N.4(b),2068]
(Ans: $x^2 + y^2 - 8x + 2y = 0$)

2. Show that the tangents to the circle $x^2 + y^2 = 100$ at the points (6,8) and (8, – 6) are perpendicular to each other. [Q.N.9(a),2068]

3. Find the equation of the circle concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through (5,4).
[Ans: $x^2 + y^2 - 8x + 12y - 49 = 0$] [Q.N. 4(b), Set 'A' 2069]

4. Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ which are perpendicular to $3x - 4y = 1$.
(Ans: $4x + 3y + 5 = 0$ and $4x + 3y - 25 = 0$) [Q.N. 9(a), Set 'A' 2069]

5. Find the equation to the circle which has the points (0, –1) and (2, 3) as ends of a diameter. [Q.N. 4(b), Set 'B' 2069]
(Ans: $x^2 + y^2 - 2x - 2y - 3 = 0$)

6. Show that the circles $x^2 + y^2 - 6x - 6y + 10 = 0$ and $x^2 + y^2 = 2$ touch each other at (1, 1). [Q.N. 9(a), Set 'B' 2069]

7. Find the equation of the circle with (0, 0) and (4, 7) as the ends of a diameter.
(Ans: $x^2 + y^2 - 4x - 7y = 0$) [Q.N. 4(b), Supp. 2069]

8. Find the equation of the tangent and normal to the circle. $x^2 + y^2 - 2x - 4y + 3 = 0$ at (2, 3) [Q.N. 9(a), Supp. 2069]
(Ans: $x + y - 5 = 0, x - y + 1 = 0$)

9. Find the equation of the circle whose two of the diameters are $x + y = 6$ and $x + 2y = 8$ and radius 10. [Q.N. 4(b), 2070 'C']
(Ans: $x^2 + y^2 - 8x - 4y - 80 = 0$)

10. Find the equations of the tangent and normal to the circle $x^2 + y^2 - 3x + 10y - 5 = 0$ at the point (4, −11) [Q.N. 9(a), 2070 'C']
(Ans: $5x - 12y - 132 = 0, 12x + 5y + 7 = 0$)

11. Find the centre and the radius of the circle $x^2 + y^2 + 4x - 6y + 4 = 0$ [Q.N. 4(b), 2070 'D']
(Ans: (− 2, 3), 3)

12. Find the value of k so that the line $4x + 3y + k = 0$ may touch the circle $x^2 + y^2 - 4x + 10y + 4 = 0$. [Q.N. 9(a), 2070 'D']
(Ans: − 18 or 32)

Unit 12 – Limit and Continuity

12.1 Limits

1. Evaluate $\lim\limits_{x \to \infty} (\sqrt{x} - \sqrt{x - 3})$ [Q.N.5(a), 2056]
(Ans: 0)

2. Determine the limit of $f(x) = \begin{cases} 2 - x^2 & \text{for } x < 2 \\ x - 4 & \text{for } x > 2 \end{cases}$ at x = 2, if it exists.
(Ans: −2) [Q.N.6(a), 2056]

3. Prove geometrically $\lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ [Q.N.12(b), 2056]

4. Evaluate : $\lim\limits_{x \to 1} \frac{x^2 + 3x - 4}{x - 1}$ [Q.N.5(a), 2057]
(Ans: 5)

5. Find the limit of the function $f(x) \begin{cases} = x^2 + 2, & x \le 5 \\ = 3x + 12, & x > 5 \end{cases}$ at x = 5 if it exists.
(Ans: 27) [Q.N.6(a), 2057]

6. Evaluate: $\lim\limits_{x \to y} \frac{\sin x - \sin y}{x - y}$ [Q.N.12(b), 2057]
(Ans: cosy)

7. Evaluate: $\lim\limits_{x \to \infty} (\sqrt{x + a} - \sqrt{x})$ [Q.N.5(a), 2058]
(Ans: 0)

8. Prove geometrically, $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$ [Q.N.12(b) (Or), 2058]

9. Evaluate: $\lim\limits_{x \to 0} \frac{1 - \cos px}{1 - \cos qx}$. [Q.N.5(a), 2059]
$\left(\text{Ans: } \frac{p^2}{q^2}\right)$

10. Does the limit of the function, $f(x) \begin{cases} = 2x + 1 & \text{for } x \; 1 \\ = 4x^2 - 1 & \text{for } x < 1 \end{cases}$ at x = 1 exist ?
[Q.N.6(a), 2059]
$\left(\text{Ans: The limit of } f(x) \text{ at } x = 1 \text{ exists and } \lim\limits_{x \to 1} f(x) = 3\right)$

11. Evaluate: $\lim_{x\to\infty} \sqrt{x}\left(\sqrt{x}-\sqrt{x-a}\right)$ [Q.N.12(b), 2059]

$\left(\text{Ans: } \frac{a}{2}\right)$

12. Does the limit of the function f (x) = $\begin{cases} x \text{ when } x > 0, \\ -x \text{ when } x < 0, \end{cases}$ exist at x = 0 ? Justify your answer. [Q.N.5(a), 2060]

13. Evaluate : $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$ [Q.N.12(b), 2060]

$\left(\text{Ans: } \frac{1}{2}\right)$

14. Evaluate : $\lim_{x\to 1} \frac{\sqrt{2x}-\sqrt{3-x^2}}{x-1}$ [Q.N.5(a), 2061]

(Ans: $\sqrt{2}$)

15. Evaluate : $\underset{x\to y}{\text{Lt}} \frac{\tan x - \tan y}{x-y}$. [Q.N.12(b), 2061]

(Ans: $\sec^2 y$)

16. Evaluate : $\underset{x\to a}{\text{Lt}} \frac{x^{2/3}-a^{2/3}}{x-a}$ [Q.N.5(a), 2062]

$\left(\text{Ans: } = \frac{2}{3a^{1/3}}\right)$

17. Prove that : $\underset{x\to 1}{\text{Lt}} \frac{x-\sqrt{2-x^2}}{2x-\sqrt{2+2x^2}} = 2$ [Q.N.12(b), 2062]

18. A function is defined as :

$f(x) = \begin{cases} 3x^2+2 & \text{if } x < 1 \\ 2x+3 & \text{if } x \geq 1 \end{cases}$

Find $\underset{x\to 1}{\text{Lt}} f(x)$. [Q.N.12(b) (Or), 2062]

(Ans: 5)

19. Show that : $\underset{x\to 2}{\text{Lt}} \frac{x^2-4}{\sqrt{3x-2}-\sqrt{x+2}} = 8$ [Q.N.5(c), 2063]

20. Find the limit of the function for $f(x) = x + 2$ when $x \geq 0$ and $f(x) = 4x + 2$ when $x < 0$ at $x = 0$. [Q.N.6(c), 2063]

(Ans: 2)

21. Evaluate : $\underset{x\to \theta}{\text{Lt}} \frac{x\text{Cos}\theta - \theta\text{Cos}x}{x-\theta}$. [Q.N.13(b) Or, 2063]

(Ans: $(\theta \sin\theta + \cos\theta)$)

22. Evaluate $\lim_{x\to 0} \frac{\text{Sin } x^\circ}{x}$ [Q.N. 5(c), 2064]

$\left(\text{Ans: } \frac{\pi}{180^\circ}\right)$

23. Evaluate $\lim_{x \to 0} \frac{\tan x - \tan y}{x - y}$ [Q.N. 13(b), 2064]

(Ans: $\sec^2 y$)

24. Evaluate : $\lim_{x \to a} \frac{\text{Sin}(x - a)}{x^2 - a^2}$. [Q.N. 5(a), 2065]

$\left(\text{Ans: } \frac{1}{2a}\right)$

25. Evaluate : $\text{Lt}_{x \to \frac{\pi}{2}} \frac{\tan x + \text{Cot}\, x}{\tan x - \text{Cot}\, x}$ [Q.N. 12(b), 2065]

(Ans: 1)

26. Evaluate : $\text{Lim}_{x \to 0} \frac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{x}$ [Q.N.5 (a), 2066]

(Ans: 0)

27. Determine the limit of

$$f(x) = \left.\begin{array}{ll} 2 - x^2 & \text{for } x \le 2 \\ = x - 4 & \text{for } x > 2 \end{array}\right\} \text{ at } x = 2, \text{ if it exists.}$$

(Ans: – 2) [Q.N.6 (a), 2066]

28. Evaluate : $\text{Lim}_{x \to \theta} \frac{x\sin\theta - \theta\sin x}{x - \theta}$. [Q.N.12 (b), 2066]

(Ans: $\sin\theta - \theta\cos\theta$.)

29. Evaluate : $\text{Lt}_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)$ 2 [Q.N.5 (a), 2067]

$\left(\text{Ans: } \frac{-1}{2}\right)$

30. Evaluate : $\text{Lt}_{x \to 0} \frac{(a + x)\sec(a + x) - a\sec a}{x}$ 4 [Q.N.12 (b), 2067]

(Ans: $a \sin a \sec^2 a + \sec a$)

31. Evaluate: $\lim_{x \to a} \frac{\sin(x-a)}{(x^2-a^2)}$ [Q.N.4(c),2068]

$\left(\text{Ans: } \frac{1}{2a}\right)$

32. Evaluate: $\lim_{x \to a} \left(\frac{\sqrt{3x} - \sqrt{2x + a}}{2(x - a)}\right)$ [Q.N.9(b),2068]

$\left(\text{Ans: } \left(\frac{1}{4\sqrt{3a}}\right)\right)$

33. Evaluate : $\lim_{x \to \infty} (\sqrt{x} - \sqrt{x - 3})$

(Ans: 0) [Q.N. 4(c), Set 'A' 2069]

34. Evaluate: $\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$ [Q.N. 9(b), Set 'A' 2069]

(Ans: $\theta\sin\theta + \cos\theta$)

35. Evaluate : $\lim_{x \to 0} \frac{1-\cos px}{1-\cos qx}$ [Q.N. 4(c), Set 'B' 2069]

$\left(\text{Ans: } \frac{p^2}{q^2}\right)$

36. Evaluate: $\lim_{x \to 2} \frac{x-\sqrt{8-x^2}}{\sqrt{x^2+12}-4}$ [Q.N. 9(b), Set 'B' 2069]

(Ans: 4)

37. Evaluate : $\lim_{x \to 0} \frac{1-\cos 6x}{x^2}$. [Q.N. 4(c), Supp. 2069]

(Ans: 18)

38. Evaluate: $\lim_{x \to a} \frac{\sqrt{3a-x}-\sqrt{x+a}}{4(x-a)}$ [Q.N. 9(b), Supp. 2069]

$\left(\text{Ans: } \frac{-1}{4\sqrt{2a}}\right)$

39. Evaluate : $\lim_{x \to 4} \frac{x^3-64}{x^2-16}$ [Q.N. 4(c), 2070 'C']

(Ans: 6)

40. Evaluate: $\lim_{x \to \theta} \frac{x\cot\theta-\theta\cot x}{x-\theta}$ [Q.N. 9(b), 2070 'C']

$\left(\text{Ans: } \cot\theta + \frac{\theta}{\sin^2\theta}\right)$

41. Evaluate : $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x-2}{\tan x-1}$ [Q.N. 4(c), 2070 'D']

(Ans: 2)

42. Evaluate: $\lim_{x \to \infty} \sqrt{x}\,(\sqrt{x}-\sqrt{x-a})$. [Q.N. 9(b), 2070 'D']

$\left(\text{Ans: } \frac{a}{2}\right)$

12.2 Continuity

1. When does a function f(x) become continuous at x = a ? Is the function f(x) defined by

$$f(x) = \begin{cases} 3+2x & -3/2 \le x < 0 \\ 3-2x & 0 \le x < 3/2 \\ -3-2x & x \ge 3/2 \end{cases}$$ continuous at $x = \frac{3}{2}$? [Q.N.12(b) (Or), 2056]

(Ans: f(x) is not continuous at $x = \frac{3}{2}$)

2. When does a function $f(x)$ become continuous at x = a ? Discuss the continuity of:

$$f(x) = \begin{cases} 2x+1, & \text{for } x < 1 \\ 2x, & \text{for } x = 1 \\ 3x, & \text{for } x > 1 \end{cases}$$ at x = 1. [Q.N.12(b) (Or), 2057]

(Ans: f(x) is discontinuous at x = 1)

3. Is the function $f(x) = \frac{x^2-9}{x-3}$ continuous at x = 3? Justify your answer.

(Ans: f(x) is discontinuous at x = 3) [Q.N.6(a), 2058]

4. Discuss the continuity of the function,

$$f(x) = \left.\begin{matrix} 2x+1 \text{ for } x<1 \\ 2x \text{ for } x=1 \\ 3x \text{ for } x>1 \end{matrix}\right\} \text{at } x=1$$

[Q.N.12(b), 2058]

(Ans: $f(x)$ is discontinuous at $x = 1$)

5. When does a function f(x) become continuous at a given point x = a?

Test the continuity of: $\left.\begin{matrix} f(x) = 2x+2 & \text{for } x<1 \\ = 2x & \text{for } x=1 \\ = 3x & \text{for } x>1 \end{matrix}\right\} \text{at } x=1$

[Q.N.12(b) (Or), 2059]

$\left(\textbf{Ans: } \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x), \textbf{ discontinuous}\right)$

6. A function is defined as $f(x) = \begin{matrix} x^2-1 & \text{when } x<1 \\ x^2+1 & \text{when } x\geq 1 \end{matrix}$

Examine whether the function is continuous or not at x = 1. **[Q.N.6(a), 2060]**

(Ans: $f(x)$ is not continuous at $x = 1$)

7. Discuss the continuity of the function : $f(x) = |x|$ at $x = 0$ **[Q.N.12(b) (Or), 2060]**

(Ans: $f(x)$ is continuous at $x = 0$)

8. Show that the function $f(x) = \begin{cases} x+2 & \text{for } x\neq 2 \\ 0 & \text{for } x=2 \end{cases}$, is not continuous at x = 2

[Q.N.6(a), 2061]

9. Discuss the continuity of the function

$$f(x) = \begin{cases} x\sin\frac{1}{x} & \text{where } x\neq 0 \\ 0 & \text{when } x=0 \end{cases}$$

[Q.N.12(b) (Or), 2061]

(Ans: $f(x)$ is continuous, at $x = 0$)

10. If the function $f(x) = \frac{1}{1-x}$ continuous at $x = 1$? **[Q.N.6(a), 2062]**

(Ans: $f(x)$ is discontinuous at $x = 1$)

11. Let a function $f(x)$ be defined by

$$\begin{aligned} f(x) &= 2-x^2 && (x<2) \\ &= 3 && (x=2) \\ &= x-4 && (x>2) \end{aligned}$$

Verify that the limit of the function exists at $x = 2$. Is the function continuous at $x = 2$? State how can you make it continuous. **[Q.N.13(b), 2063]**

(Ans: $f(x)$ is not continuous at $x = 2$)

12. Why the function $f(x) = \text{Sin}\,\frac{1}{x}$ is not continuous at x = 0 ? **[Q.N. 6(c), 2064]**

(Ans: not continuous)

13. Show that the function $f(x) = \dfrac{\text{Sin}^2 ax}{x^2}, \ (x\neq 0)$

$= 1, \ (x=0)$

is discontinuous at x = 0.

Redefine the function is such a way that it becomes continuous at x = 0.

[Q.N. 13(b)(or), 2064]

14. Discuss the continuity of the function $\frac{x^2-9}{x-3}$ and point out the discontinuity if exists. **[Q.N. 6(a), 2065]**

(Ans: discontinuous)

15. Test the continuity of the function

$$f(x) = \begin{cases} x, \text{ when } 0 \le x < \frac{1}{2} \\ 1, \text{ when } x = \frac{1}{2} \\ 1 - x, \text{ when } \frac{1}{2} < x < 1 \end{cases}$$

at $x = \frac{1}{2}$. [Q.N. 12(b, or), 2065]

(Ans: discontinuous)

16. A function is defined as follows : $f(x) = -x$ when $x<0$
$= x$ when $0<x<1$
$= 2 - x$ when $x \ge 1$

show that it is continuous at x = 0 and x = 1 [Q.N.12 (b) (or), 2066]

17. Test the continuity of $f(x) = x + 2$ when $x \neq 2$,
$= 4$ when $x = 2$; at $x = 2$.

(Ans: Continuous) 2 [Q.N.6 (a), 2067]

18. Discuss the continuity of the function :

$f(x) = x \sin 1/x$ when $x \neq 0$

$= 0$ when $x = 0$; at $x = 0$.

(Ans: Continuous) 4 [Q.N.12 (b)Or, 2067]

19. A function f (x) is defined as follows:

$$f(x) = \begin{cases} 2x+1 & \text{for } x<1 \\ 2 & \text{for } x=1 \\ 3x & \text{for } x>1 \end{cases}$$

Is the function continuous at x = 1 ? If not, can it be made continuous at x = 1 ? [Q.N.9(b)(Or),2068]

(Ans: Not continuous)

20. Define continuity of a function at a point. A function is defined as follows:

$$f(x) = \begin{cases} \dfrac{2x^2 - 18}{x - 3} & \text{for } x \neq 3 \\ K & \text{for } x = 3 \end{cases}$$

find the value of k so that f(x) is continuous at x = 3.

(Ans: 10) [Q.N. 9(b)(Or), Set 'A' 2069]

21. Show that the funcation

$$f(x) = \begin{cases} x & \text{when } 0 \le x < \frac{1}{2}, \\ 1 & \text{when } x = \frac{1}{2} \\ 1 - x & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

is discontinuous at $x = \frac{1}{2}$. Also, write how it could be made continuous?

[Q.N. 9(b)(Or), Set 'B' 2069]

22. Let a function f(x) be defined by $f(x) = \begin{cases} 2-x^2 & \text{for } x < 2 \\ 3 & \text{for } x = 2 \\ x-4 & \text{for } x > 2 \end{cases}$

Show that the limit of the function f(x) exists as x = 2. is the function f(x) continuous at x=2? If not, how would you make it continuous?

[Q.N. 9(b)(Or), Supp. 2069]

23. A function f(x) is defined as follows: $f(x) = \begin{cases} 3+2x & \text{for } -\frac{3}{2} \le x < 0 \\ 3-2x & \text{for } 0 \le x < \frac{3}{2} \\ -3-2x & \text{for } x \ge \frac{3}{2} \end{cases}$

Show that f(x) is continuous at x = 0 but discontinuous at $x = \frac{3}{2}$.

[Q.N. 9(b)(Or), 2070 'C']

24. A function f(x) is defined below:

$f(x) = \begin{cases} kx+3 & \text{for } x \ge 2 \\ 3x-1 & \text{for } x < 2 \end{cases}$

Find the value of k so that f(x) is continuous at x = 2.

(Ans: 1) **[Q.N. 9(b)(Or), 2070 'D']**

Unit 13 – The Derivatives

1. Find dy/dx of $y = e^{5x} \sin(\log x)$. **[Q.N.5(c), 2056]**

$\left(\textbf{Ans: } e^{5x}\left[\frac{\cos(\log x)}{x} + 5\sin(\log x)\right]\right)$

2. Find, from the first principles the derivative of : $y = \frac{1}{\sqrt{ax+b}}$ **[Q.N.13(a), 2056]**

$\left(\textbf{Ans: } -\frac{a}{2(ax+b)^{\frac{3}{2}}}\right)$

3. Find $\frac{dy}{dx}$ of $y = e^{\sin(\log x)}$. **[Q.N.5(c), 2057]**

$\left(\textbf{Ans: } \frac{1}{x} e^{\sin(\log x)} . \cos(\log x)\right)$

4. Find, from the first principles the derivative of $y = \sqrt{\sin 2x}$ **[Q.N.13(a), 2057]**

$\left(\textbf{Ans: } \frac{\cos 2x}{\sqrt{\sin 2x}}\right)$

5. Find $\frac{dy}{dx}$ of x = a sint, y = a cost. **[Q.N.5(c), 2058]**

(Ans: – tant)

6. Find, from definition, the derivatives of sin2x **[Q.N.13(a), 2058]**

(Ans: 2cos2x)

7. Find $\frac{dy}{dx}$ of x = a sint, y = a cost. **[Q.N.5(c), 2059]**

(Ans: – tant)

8. Find from definition the derivatives of sin2x. [Q.N.13(a), 2058]
(Ans: 2cos2x)

9. Find $\frac{dy}{dx}$ of x = a sint, y = a cost. [Q.N.13(a)Or, 2058]
(Ans: – tant)

10. Find from the first principles the derivative of $\frac{1}{\sqrt{3x-4}}$. [Q.N.13(a), 2059]
$\left(\textbf{Ans: } \frac{-3}{2\,(3x-4)^{3/2}}\right)$

11. Find $\frac{dy}{dx}$ when x= 2a tan and y = a $\sec^2\theta$ [Q.N.5(c), 2060]
(Ans: tanθ)

12. Find $\frac{dy}{dx}$ from first principle y = $\sqrt{\tan x}$ [Q.N.13(a), 2060]
$\left(\textbf{Ans: } \frac{\sec^2 x}{2\sqrt{\tan x}}\right)$

13. Differentiate Sinx with respect to tanx. [Q.N.5(c), 2061]
(Ans: $\cos^3 x$)

14. Find $\frac{dy}{dx}$ from first principle when y = x + $\sqrt{x}$ [Q.N.13(a), 2061]
$\left(\textbf{Ans: } 1+\frac{1}{2\sqrt{x}}\right)$

15. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\frac{2x}{1-x^2}$ [Q.N.5(c), 2062]
$\left(\textbf{Ans: } \frac{1}{1+x^2}\right)$

16. Find from definition, the derivative of $\sqrt{\tan x}$. Show that the rectangle of largest possible area for a given perimeter is a square. [Q.N.13(a), 2062]
$\left(\textbf{Ans: } \frac{\sec^2 x}{2\sqrt{\tan x}}\right)$

17. Find $\frac{dy}{dx}$ when $y = \text{Sin}^{-1}(3x-4x^3)$ [Q.N.5(b), 2063]
$\left(\textbf{Ans: } \frac{3}{\sqrt{1-x^2}}\right)$

18. Find, from definition, the derivative of $\frac{1}{\sqrt{x+2}}$ [Q.N.12(a) Or, 2063]
$\left(\textbf{Ans: } \frac{-1}{2\,(x+2)^{3/2}}\right)$

19. Find $\frac{dy}{dx}$ where $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ [Q.N. 5(b), 2064]
$\left(\textbf{Ans: } \frac{1}{1+x^2}\right)$

20. Find from definition the derivative of $\text{Cos}^2 x$. [Q.N. 12(b)(or), 2064]
(Ans: – cos2x)

21. Find $\frac{dy}{dx}$, when
y = Sinθ and θ = $5x^2 - 6x + 2$. [Q.N. 5(c), 2065]
(Ans: $(10x-6)\cos(5x^2-6x+2)$)

22. Find from first principles, the derivative of $\sqrt{\text{Sin } 2x}$. [Q.N. 13(a), 2065]

$\left(\text{Ans: } \frac{\cos 2x}{\sqrt{\sin 2x}}\right)$

23. Find $\frac{dy}{dx}$ if $ax^2 + 2hxy + by^2 = 1$ [Q.N.5 (c), 2066]

$\left(\text{Ans: } \frac{-ax + hy}{hx + by}\right)$

24. Find from first principles the derivatives of sin2x.
(Ans: 2cos2x) [Q.N.13 (a), 2066]

25. Find the derivative of $\tan^{-1} \frac{\sin 2x}{1 + \cos 2x}$ 2 [Q.N.5 (c), 2067]

(Ans: 1)

26. Find from definition, the derivative of $\frac{1}{\sqrt{x}}$. 4 [Q.N.13 (a), 2067]

$\left(\text{Ans: } \frac{1}{2x\sqrt{x}}\right)$

27. Find the derivative of $\frac{1}{x - \sqrt{a^2+x^2}}$ [Q.N.5(a),2068]

$\left(\text{Ans: } \frac{-1}{a^2}\left(1 + \frac{x}{\sqrt{a^2+x^2}}\right)\right)$

28. Find from first principle, the derivative of sin4x. [Q.N.10(a),2068]
(Ans: 4 cos 4x)

29. Find $\frac{dy}{dx}$ if $x^3 + y^3 - 3axy = 0$.

$\left(\text{Ans: } \frac{ay - x^2}{y^2 - ax}\right)$ [Q.N. 5(a), Set 'A' 2069]

30. Find from first principles the derivative of $\sqrt{2x+3}$.

$\left(\text{Ans: } \frac{1}{\sqrt{2x+3}}\right)$ [Q.N. 10(a), Set 'A' 2069]

31. Find $\frac{dy}{dx}$ when $y = \frac{1}{\sec x - \tan x}$. [Q.N. 5(a), Set 'B' 2069]

(Ans: $\sec x \tan x + \sec^2 x$)

32. Find from first principles the derivative of $f(x) = \frac{1}{\sqrt{x+a}}$ [Q.N. 10(a), Set 'B' 2069]

$\left(\text{Ans: } \frac{-1}{2(x+a)^{\frac{3}{2}}}\right)$

33. Find $\frac{dy}{dx}$ if $x = 2a \tan \theta$ and $y = a \sec^2\theta$. [Q.N. 5(a), Supp. 2069]

(Ans: $\tan \theta$)

34. Find from first principle, the derivative of tan3x. [Q.N. 10(a), Supp. 2069]
(Ans: $3\sec^2 3x$)

35. Find $\frac{dy}{dx}$ when $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$. [Q.N. 5(a), 2070 'C']

$\left(\text{Ans: } \frac{t^2+1}{t^2-1}\right)$

36. Find from first principles the derivative of $\sqrt{2-3x}$.

$\left(\text{Ans} - \frac{3}{2\sqrt{2-3x}}\right)$ [Q.N. 10(a), 2070 'C']

37. Find $\frac{dy}{dx}$ when $x - y = \tan xy$. [Q.N. 5(a), 2070 'D']

$\left(\text{Ans: } \frac{1 - y\sec^2 xy}{1 + x\sec^2 xy}\right)$

38. Find from first principles, the derivative of $\sqrt{1+x}$. [Q.N. 10(a), 2070 'D']

$\left(\text{Ans: } \frac{1}{2\sqrt{1+x}}\right)$

Unit 14 – Applications of Derivatives

14.1 Increasing and Decreasing Function

1. Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$. [Q.N.5(c),2068]

   (Ans: Decreasing at 2/5 , Increasing at 5/2)

2. For any curve $y = f(x)$, what do $f'(x) > 0$ and $f'(x) < 0$ represent? [Q.N. 5(c), Set 'B' 2069]

3. Test the increasing and decreasing of the function $f(x) = x^2 - 3x + 4$ at the points $x = 2$ and $x = 1$. [Q.N. 5(c), Supp. 2069]

   (Ans: Increasing at x = 2, decreasing at x = 1)

4. Find the interval in which the function $f(x) = 3x^2 - 6x + 5$ is increasing or decreasing. [Q.N. 5(c), 2070 'C']

   (Ans: Increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$)

14.2 Extrema of a Function

1. Determine where the graph is concave upwards or concave downwards for $f(x) = x^4 - 8x^3 + 18x^2 - 24$. Also find the point of inflection. [Q.N.13(a) (Or), 2056]

   Ans: Concave upwards for x<1 and x>3; Concave downwards for 1<x<3; Points of inflection x = 1, x = 3

2. Find the maximum and minimum values of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$. Also find the point of inflection. [Q.N.13(a) (Or), 2057]

   Ans: $y_{max.} = 3\frac{1}{2}$, $y_{min.} = -12\frac{1}{2}$ and Point of inflection $x = \frac{1}{2}$

3. Show that the rectangle of largest possible area for a given perimeter is a square. [Q.N.13(a) (Or), 2058]

4. Show that the rectangle of largest possible area for a given perimeter, is a square. [Q.N.13(a) (Or), 2059]

5. Find the maximum and minimum value of the function $x^3 - 3x^3 + 6x + 5$, if exist. Also, find the point of inflexion. [Q.N.13(a) (Or), 2060]

   (Ans: Neither maximum nor minimum and Point of inflection is x = 1)

6. A man wishes to fence a rectangular garden with 256 m. fencing material. Find the maximum area he can enclose. [Q.N.13(a) (Or), 2061]

   (Ans: Maximum value of A = 4096m²)

7. Calculate the maximum and minimum values of $x^3 - 3x^2 - 9x + 27$. [Q.N.12(a), 2063]

   (Ans: max = 32, min = 0)

8. Find the maximum area of a rectangular plot of land which can be enclosed by a rope of length 60 metres. **[Q.N. 12(b), 2064]**

(Ans: 225 m^2)

9. Show that the rectangle of largest possible area for a given perimeter is a square. **[Q.N. 13(a, or), 2065]**

10. Using derivatives, find two numbers whose sum is 10 and sum of whose squares is minimum. **[Q.N.13 (a) (or), 2066]**

(Ans: 5, 5)

11. A man wishes to fence a rectangular garden with 256 meter fencing material. Find the maximum area he can enclose.

(Ans: 4096 m^2) **4 [Q.N.13 (a)Or, 2067]**

12. List the criteria for the function y = f(x) to have local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection. **[Q.N.15,2068]**

(Ans: Max: 9.75, Min: 3, Pt. of inflection x = 5/4)

13. What are the criteria for a function y = f(x) to have the local maxima and local minima at a point? Find the local maxima and local minima of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1,2). Also find the point of inflection.

$\left(\text{Ans: Max} : 3\frac{1}{2}, \text{Min}: -12\frac{1}{2}, \text{point of inf} = \frac{1}{2}\right)$ **[Q.N. 15, Set 'A' 2069]**

14. Find the maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x - 2$. Also, find the point of inflection, if any.

[Q.N. 15, Set 'B' 2069]

(Ans: (Max : 2 at x = 1, Min : –2 at x = 3) x = 2 is point of inflection)

15. What are the criteria for the graph of the function y=f(x) to have concave upward and concave downward? Determine where the graph is concave upward and where it is concave downward of the function. $f(x) = x^4 - 8x^3 + 18x^2 - 24$. **[Q.N. 15, Supp. 2069]**

(Ans: Concave upward for x < 1, x > 3, concave downward for 1 < x < 3)

16. List the criteria for the function y = f(x) to have the local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection. **[Q.N. 15, 2070 'C']**

$\left(\text{Ans: Min: 3 at x= 2; Max} : 9.75 \text{ at } x = \frac{1}{2}, \text{Point of inflection } x = \frac{5}{4}\right)$

17. Write the criteria for the function y = f(x) to have the local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 2x^3 - 9x^2 - 24x + 3$. Also find the point of inflection. **[Q.N. 15, 2070 'D']**

$\left(\text{Ans: Max. value} = 16, \text{Min} = -109, \text{Point of inflection} = \frac{3}{2}\right)$

14.3 Derivative as a Rate Measure

1. A Spherical ball of salt dissolving in water decreases its volume at the rate of 0.75cm^3/min. Find the rate at which the radius of the salt is decreasing when its radius is 6cm. **[Q.N.15(Or),2068]**

(Ans: 1.657×10^{-3} cm/min)

2. The side of a square sheet is increasing at the rate of 5cm/min. At what rate is the area increasing when the side is 12 cm. long?

[Ans: 120 cm^2 /min] **[Q.N. 5(c), Set 'A' 2069]**

3. The volume of a spherical balloon is increasing at the rate of 25 cubic cm/sec. Find the rate of change of its surface at the instant when its radius is 5 cm.

(Ans; 10 sq. cm/sec.) **[Q.N. 15(Or), Set 'B' 2069]**

4. Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 10cm/sec and that of the outer circle at the rate of 7cm/sec. At a certain time, the redei of the inner and the outer circle are respectively 24 cm and 30cm. At what time, is the area between the circles increasing or decreasing? How fast? **[Q.N. 15(Or), Supp. 2069]**

(Ans: Decreasing at -264π sq. cm/sec)

5. A spherical ball of salt is dissolving in water in such a way that the rate of decrease in volume at any instant is proportional to the surface. Prove that the radius is decreasing at the constant rate. **[Q.N. 15(Or), 2070 'C']**

6. A stone thrown into a pond produces circular ripples which expands from the point of impact. If the radius of the ripple increases at the rate of 3.5cm/sec, find how fast is the area growing when the radius is 15cm. $\left(\pi = \frac{22}{7}\right)$

(Ans: 330 cm^2/sec) **[Q.N. 5(c), 2070 'D']**

Unit 15 – Antiderivatives and its Applications

15.1 Antiderivative

1. Integrate: $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx.$ **[Q.N.6(c), 2056]**

$\left(\textbf{Ans: } \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C\right)$

2. Evaluate : $\int \frac{dx}{\sqrt{a^2+x^2}}$ **[Q.N.13(b), 2056]**

(Ans: $\log(x+\sqrt{a^2+x^2})+C$)

3. Integrate: $\int x \sin x \, dx.$ **[Q.N.6(b), 2057]**

(Ans: $\sin x - x\cos x + C$)

4. Find the value of : $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos t \, dt.$ **[Q.N.6(c), 2057]**

(Ans: $\sqrt{3}$)

5. Integrate $\int x^2 \sin x \, dx.$ **[Q.N.13(b), 2057]**

(Ans: $-x^2\cos x + 2x\sin x + 2\cos x + C$)

6. Integrate $\int \log x \, dx$ **[Q.N.6(c), 2058]**

(Ans: $x\log x - x + C$)

7. Evaluate: $\int x^2 e^{ax} dx$ **[Q.N.13(b), 2058]**

$\left(\textbf{Ans: } \frac{x^2e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{x}}{a^3} + C\right)$

8. Integrate $\int \sec x \, dx.$ **[Q.N.6(b), 2059]**

(Ans: $\log(\sec x + \tan x) + C$)

9. Evaluate $\int_1^2 \frac{\text{Sin}(\log x)}{x}\,dx$. [Q.N.6(c), 2059]

(Ans: $1 + \cos(\log 2)$)

10. Evaluate $\int e^x \cos x\,dx$. [Q.N.13(b), 2059]

$\left(\text{Ans: } \frac{1}{2} e^x(\cos x + \sin x) + C\right)$

11. Evaluate : $\int \text{Sin}^2 2x\,dx$. [Q.N.6(b), 2060]

$\left(\text{Ans: } \frac{x}{2} - \frac{\sin 4x}{8} + C\right)$

12. Evaluate : $\int x \sin^2 x\,dx$ [Q.N.13(b), 2060]

$\left(\text{Ans: } \frac{x^2}{4} - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + C\right)$

13. Evaluate : $\int \frac{1}{x} \cos(\log x)\,dx$. [Q.N.6(b), 2061]

(Ans: $\sin(\log x) + C$)

14. Integrate : $\int \text{Sec}^3 x\,dx$. [Q.N.13(b), 2061]

$\left(\text{Ans: } \frac{1}{2}[\sec x . \tan x + \log(\sec x + \tan x)] + C\right)$

15. Evaluate : $\int x \,\text{Sin} x\,dx$ [Q.N.6(b), 2062]

(Ans: $-x \cos x + \sin x + C$)

16. Evaluate : $\int_0^{-1} \frac{dx}{4 - x^2}$ [Q.N.13(b) (Or), 2062]

$\left(\text{Ans: } \frac{-\pi}{6}\right)$

17. Integrate : $\int \sec x\,dx$ [Q.N.6(b), 2063]

(Ans: $\log(\sec x + \tan x) + C$)

18. Evaluate : $\int_0^2 \frac{x\,dx}{\sqrt{x^2 + 4}}$ [Q.N.13(a), 2063]

(Ans: $2(\sqrt{2} - 1)$)

19. Integrate = $\int \text{cosec}\, x\,dx$ [Q.N. 6(b), 2064]

(Ans: $\log(\text{cosec} x - \cot x) + c$)

20. Evaluate $\int_1^2 \frac{\text{Sin}(\log x)}{x}\,dx$ [Q.N. 13(a), 2064]

(Ans: $1 + \cos(\log 2)$)

21. Evaluate : $\int \log x\,dx$ [Q.N. 6(b), 2065]

(Ans: $x\log x - x + c$)

22. Evaluate : $\int e^{ax} \text{Cos}\, bx\,dx$ [Q.N. 13(b), 2065]

$\left(\text{Ans: } \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + c\right)$

23. Evaluate : $\int \frac{dx}{\sqrt{2x+1}-\sqrt{2x-3}}$ [Q.N.6 (b), 2066]

$\left(\text{Ans: } \frac{1}{12}\left[(2x+1)^{\frac{3}{2}}+(2x-3)^{\frac{3}{2}}\right]+C\right)$

24. Evaluate : $\int_1^3 \frac{x\,dx}{1+x^2}$ [Q.N.13 (b), 2066]

(Ans: $\log\sqrt{5}$)

25. Evaluate : $\int \frac{dx}{1+\sin x}$ 2 [Q.N.6 (b), 2067]

$\left(\text{Ans: } \frac{-2}{\tan\frac{x}{2}}+C\right)$

26. Evaluate : $\int \sec^3 x\,dx$ 4 [Q.N.13 (b), 2067]

$\left(\frac{1}{2}[\sec x\tan x+\log(\sec x+\tan x)]+C\right)$

27. Evaluate: $\int\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$ [Q.N.5(b),2068]

$\left(\text{Ans: } e^{x+\frac{1}{x}}+c\right)$

28. Evaluate: $\int \cot x\,(\log\sin x)^3 dx$

$\left(\text{Ans: } \frac{1}{4}(\log x \sin x)^4+c\right)$ [Q.N. 5(b), Set 'A' 2069]

29. Evaluate: $\int \frac{dx}{\sin^2 x.\cos^2 x}$ [Q.N. 5(b), Set 'B' 2069]

(Ans: $-\cot 2x + c$)

30. Evaluate : $\int x \sin ax\,dx$. [Q.N. 5(b), Supp. 2069]

$\left(\text{Ans: } \frac{-\cos ax}{a}+\frac{1}{a^2}\sin ax+c\right)$

31. Evaluate: $\int \frac{1}{\sqrt{2x+1}-\sqrt{2x-3}}dx$. [Q.N. 5(b), 2070 'C']

$\left(\text{Ans: } \frac{1}{12}[(2x+1)^{3/2}+(2x-3)^{3/2}]+C\right)$

32. Evaluate: $\int \frac{1}{x}\sin(\log x)\,dx$. [Q.N. 5(b), 2070 'D']

(Ans: $-\cos(\log x)+c$)

15.2 Area Between two Curves

1. Find the area bounded by the axis of x and the curve $y = 4x^3$ and the ordinates at $x = 2$ and $x = 4$. [Q.N.6(b), 2056]
   (Ans: 240 sq. units)
2. Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$. [Q.N.13(b) (Or), 2056]
   (Ans: $\frac{16}{3}$ sq. units)
3. Find the area of the circle, $x^2 + y^2 = 25$. [Q.N.13(b) (Or), 2057]
   (Ans: 25π sq. units)
4. Find the area enclose by the curve $y = 3x$, the x – axis and ordinates at $x = 0$ and $x = 4$. [Q.N.6(b), 2058]
   (Ans: 24 sq. unit)
5. Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$ [Q.N.13(b) (Or), 2058]
   (Ans: $\frac{16a^2}{3}$ sq. units)
6. Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$. [Q.N.13(b) (Or), 2059]
   (Ans: $\frac{16}{3}$ sq. unit)
7. Find the area under the curve $y = x^2$ bounded by x-axis, and between the ordinates $x = 0$ and $x = a$. [Q.N.6(c), 2060]
   $\left(\text{Ans: } \frac{a^3}{3}\right)$
8. Find using method of integration the area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$. [Q.N.13(b) (Or), 2060]
   (Ans: $\frac{16}{3}$ sq. units)
9. Find the area bounded by curves $y = 3x^2$, $x = 1$ and $x = 3$. [Q.N.6(c), 2061]
   (Ans: 26 sq. unit)
10. Using integration, find the area of the circle $x^2+y^2=a^2$. [Q.N.13(b) (Or), 2061]
    (Ans: πa^2 sq. units)
11. Find the area bounded by the x- axis and the curve and $y = \log(1+x)$ and ordinates $x = 0$ and $x = 1$ [Q.N.6(c), 2062]
    (Ans: $2 \log 2 - 1$)
12. Find the area of the ellipse : $\frac{x^2}{9} + \frac{y^2}{16} = 1$ [Q.N.13(b), 2062]
    (Ans: 12π sq. units)
13. Find the area bounded by the x – axis and the following curve and ordinates $xy = 8$; $x = 3$, $x = 8$ [Q.N.6(a), 2063]
    $\left(\text{Ans: } 8 \log \frac{8}{3} \text{ sq. units}\right)$
14. Find the area of the circle $x^2 + y^2 = 25$, using method of integration.
    (Ans: 25π sq. units) [Q.N.13(a) Or, 2063]

15. Find the area bounded by the x-axis and the following curve and ordinates $y = \log x$, $x = 1$, $x = e$ [Q.N. 6(a), 2064]
(Ans: 1 sq. unit)

16. Find the area of the circle $x^2 + y^2 = 9$ using method of integration. [Q.N. 13(a)(or), 2064]
(Ans: 9π sq. units)

17. Find the area of the region bounded by the curve $y = e^x$, the x-axis and the ordinates $x = 1$; $x = 2$. [Q.N. 6(c), 2065]
(Ans: $(e^2 - e)$ sq. units)

18. Using method of integration, find the area under the curve $x^2 + y^2 = a^2$.
(Ans: πa^2 sq. units) [Q.N. 13(b, or), 2065]

19. Find the area bounded by the curve $y = \sin x$, $x = 0$, $x = \pi$.
(Ans: 2sq. unit) [Q.N.6 (c), 2066]

20. Find the area of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$
(Ans: 6π sq. unit) [Q.N.13 (b)(or), 2066]

21. Find the area under the curve $y = 2\sqrt{x}$ between $x = 0$ and $x = 1$.
(Ans: $\frac{4}{3}$ sq. unit) 2 [Q.N.6 (c), 2067]

22. Find the area under the curves $\frac{x^2}{16} + \frac{y^2}{25} = 1$ using method of integration.
(Ans: 20π sq. unit) 4 [Q.N.13 (b)Or, 2067]

23. Find the area bounded by the curve $y^2 = 4ax$ and the line $x = a$.
(Ans: $\frac{8a^2}{3}$ sq. units.) [Q.N.10(b),2068]

24. Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$.
(Ans: $\frac{16}{3}$ sq. units.) [Q.N. 10(b), Set 'A' 2069]

25. Find the area of the region bounded by the curves.
$x^2 + 4y$ and $x = y$ [Q.N. 10(b), Set 'B' 2069]
(Ans: $\frac{8}{3}$ sq. units)

26. Find the area bounded by y-axis, the curve.
$x^2 = 4a(y - 2a)$ and $y = 6a$. [Q.N. 10(b), Supp. 2069]
(Ans: $\frac{32}{3}a^2$ sq unit)

37. Find the area bounded by y-axis, the curve $x^2 = 4a(y - 2a)$ and $y = 6a$.
(Ans: $\frac{32}{3}a^2$ sq. unit) [Q.N. 10(b), 2070 'C']

38. Find the area enclosed by the axis of x and the curve $y = 3x - 5x^2$. [Q.N. 10(b), 2070 'D']
(Ans: $\frac{9}{50}$ sq. units)