30. Show that the points A, B and C with position vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-7\vec{j} + 10\vec{k}$ respectively are collinear. [Q.N. 10(a), 2058]

31. Show that the vectors $2\vec{i} + 3\vec{j} - 8\vec{k}$ and $2\vec{i} + 4\vec{j} + 2\vec{k}$ are orthogonal. [Q.N. 3(b), 2057]

32. If the position vector of M and N are $3\vec{i} + \vec{j} - 3\vec{k}$ and $4\vec{i} - 2\vec{j} + \vec{k}$ respectively, find $\vec{MN}$ and determine its direction cosines. [Q.N. 10(a), 2057]

$\left(\text{Ans: } \vec{MN} = \vec{i} - 3\vec{j} + 4\vec{k}, \left(\frac{1}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)\right)$

33. A B C D E F is a regular hexagon. Express $\vec{AC}$ and $\vec{AD}$ in terms of $\vec{AB}$ and $\vec{BC}$.

$\left(\text{Ans: } \vec{AC} = \vec{AB} + \vec{BC}, \vec{AD} = 2\vec{BC}\right)$ [Q.N. 4(a), 2057]

6.2 Product of Vectors

1. Find the angle between the vectors $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} - 3\vec{j} - 5\vec{k}$.
(Ans: 90°) 2[Q.N.3(c), 2072'C']
2. Define Vector product of two Vectors. Prove by Vector method:
$\sin(A + B) = \sin A\cos B + \cos A\sin B$. 6[Q.N.10, 2072'C']
3. If $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$, prove that $|\vec{a}| = |\vec{b}|$. 2[Q.N.3(c), 2072'D']
4. Define Vector product of two Vectors. Prove by Vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 6[Q.N.10, 2072'D']
5. If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\vec{PQ}$ and its direction cosines. 2[Q.N.2(c), 2072'E']

$\left(\text{Ans: } 4\vec{i} - 5\vec{j} + 11\vec{k}, \left(\frac{4}{9\sqrt{2}}, \frac{11}{9\sqrt{2}}\right)\right)$

6. Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$.
(Ans: 20.5 sq. unit) 2[Q.N.3(c), 2072'E']
7. Define scalar product of two vectors. Give the geometrical interpretation of the scalar product of two vectors. In any triangle prove vectorically that $a^2 = b^2 + c^2 - 2bc\cos A$. [Q.N.10, 2072'E']
8. For what value of m is the pair of vectors $\vec{i} - 2\vec{j} + 4\vec{k}$ and $2\vec{i} - 7\vec{j} + m\vec{k}$ orthogonal ?
(Ans: m = 3) 2 [Q.N. 3(c), Set 'C' 2071]
9. Define vector product of two vectors. Prove by vector method that
$\sin(A + B) = \sin A\cos B + \cos A\sin B$. 6 [Q.N. 10, Set 'C' 2071]
10. Find a unit vector perpendicular to each of the vectors
$3\vec{i} + \vec{j} + 2\vec{k}$ and $2\vec{i} - 2\vec{j} + 4\vec{k}$. 2 [Q.N. 3(c), Set 'D' 2071]

$\left(\text{Ans: } \frac{1}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}\right)$

11. Define scalar product of two vectors. Prove by vector method that:
$\cos(A + B) = \cos A\cos B - \sin A\sin B$. 6 [Q.N. 10, Set 'D' 2071]

12. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, prove that $\vec{a}$ is perpendicular to $\vec{b}$ 2 [Q.N. 3(c), 2070 'C']

13. Define vector product of two vectors. Using vector method, prove that:
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
6[Q.N. 10, 2070 'C']

14. Find the since of the angle between the two vectors
$2\vec{i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$. 2 [Q.N. 3(c), 2070 'D']
$\left(\text{Ans: } \sqrt{\frac{155}{156}}\right)$

15. Define scalar product of two vectors. 6[Q.N. 10, 2070 'D']
Prove by vector method that : $\cos(A - B) = \cos A \cos B + \sin A \sin B$

16. If $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 0$, prove that $|\vec{a}| = |\vec{b}|$. 2 [Q.N. 3(c), Supp. 2069]

17. Define vector product of two vectors and geometrically interpret it. Also show that $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ where $\vec{a}$ and $\vec{b}$ are any two non zero vectors.
6 [Q.N. 10, Supp. 2069]

18. Find the area of the parallelogram determined by the vectors $\vec{i} + 2\vec{j} + 3\vec{k}$ and $-3\vec{i} - 2\vec{j} + \vec{k}$ [Q.N. 3(c), Set 'A' 2069]
(Ans: $6\sqrt{5}$ sq uints)

19. Define scalar product of two vectors. Prove by the method of vectors that:
$\cos(A-B) = \cos A \cos B + \sin A \sin B$. 6 [Q.N. 10, Set 'A' 2069]

20. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find the cosine of the angle between the two vectors. [Q.N. 3(c), Set 'B' 2069]
$\left(\text{Ans: } \frac{1}{2}\right)$

21. Define vector product of two vectors. Interpret the vector product of two vectors geometrically. Prove by vector method that:
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
[Q.N. 10, Set 'B' 2069]

22. Find the area of the triangle determined by the vectors
$3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$ [Q.N. 3(b), 2068]
[Ans: 20.5 sq. units]

23. Using vector method prove that: $c^2 = a^2 + b^2 - 2ab\cos C$. [Q.N. 11(b), 2068]

24. Given $\vec{a} = (3, 1, 2)$ and $\vec{b} = (2, -2, 4)$, find the projection of $\vec{a}$ on $\vec{b}$.
(Ans: $\sqrt{6}$) [Q.N. 3(b), 2067]

25. Prove by vector method : $\cos(A+B) = \cos A \cos B - \sin A \sin B$. [Q.N. 11(a), 2067]

26. For what value of m are the vectors $\vec{i} - 2\vec{j} + 4\vec{k}$ and $2\vec{i} + 7\vec{j} + m\vec{k}$ orthogonal?
(Ans: 3) [Q.N. 3(b), 2066]

27. Use vector method to prove that, in any triangle ABC, $a = b\cos C + c\cos B$.
[Q.N. 11(a), 2066]

28. Find the value of r if the vectors $3\vec{i} - \vec{j} - 2\vec{k}$ and $2\vec{i} - 2\vec{j} + r\vec{k}$ are orthogonal.
(Ans: 4) [Q.N.4(a), 2065]

29. By using vectors, prove that in any ΔABC,
$$\frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } C}{c}$$
[Q.N.11(a), 2065]

30. Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$. *(Ans: 41 sq. units)* [Q.N. 3(b), 2064]

31. Using vector method, prove in any triangle, that :
$b^2 = c^2 + a^2 - 2ca\ CosB$ [Q.N. 11(a), 2064]

32. If $\vec{i}$, $\vec{j}$, $\vec{k}$ are three mutually perpendicular unit vectors and $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - 3\vec{j} - \vec{k}$, find the cosine of the angle between the two vectors. [Q.N. 4(a), 2063]
$\left(Ans.: \cos^{-1}\sqrt{\frac{7}{12}}\right)$

33. Using vector method, prove in any triangle that :
$a = b\ CosC + c\ CosB$ [Q.N. 11(a), 2063]

34. Find the area of the parallelogram determined by the vectors
$\vec{i} + 2\vec{j} + 3\vec{k}$ and $-3\vec{i} - 2\vec{j} + \vec{k}$ [Q.N. 3(b), 2062]
(Ans.: $6\sqrt{5}$ sq. unit)

35. Prove vectorically that : $Cos(A-B) = CosA\ Cos\ B + SinA\ SinB$ [Q.N. 11(a), 2062]

36. Find a unit vector perpendicular to $2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{i} + \vec{j} - 2\vec{k}$.
$\left(Ans: \frac{-5}{\sqrt{35}}\vec{i} + \frac{3}{\sqrt{35}}\vec{j} - \frac{1}{\sqrt{35}}\vec{k}\right)$ [Q.N. 3(b), 2061]

37. If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ is the angle between them. Show that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$ [Q.N. 4(a), 2061]

38. Prove, in any triangle, by vector method that :
$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$ [Q.N. 11(a), 2061]

39. Find the cosine of the angle between the vectors
$2\vec{i} + \vec{j} + \vec{k}$ and $4\vec{i} + 3\vec{j} + 5\vec{k}$ [Q.N. 3(b), 2060]
$\left(Ans: \frac{8}{5\sqrt{3}}\right)$

40. Prove by vector method: $\sin(A - B) = \sin A \cos B - \cos A \sin B$. [Q.N. 11(a), 2060]

41. Find the angle between two vectors $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$.
(Ans: 60°) [Q.N. 4(a), 2059]

42. Prove by vector method: $Sin(A+B) = Sin\ A\ Cos\ B + Cos\ A\ SinB$. [Q.N. 11(a), 2059]

43. Show that the area of the parallelogram determined by:
$\vec{i} + \vec{j} - 3\vec{k}$ and $-\vec{i} - 2\vec{j} - 3\vec{k}$ is $\sqrt{118}$ sq. units [Q.N. 3(b), 2058]

44. Prove by vector method.
$\cos(A-B) = \cos A \cos B + \sin A.\ \sin B.$ [Q.N. 11(a), 2058]

45. Prove by vector method
$Cos(A - B) = CosA\ CosB + SinA\ SinB$ [Q.N. 11(a), 2057]

Unit 7: Derivative and its Application

7.1 Continuity and differentiability

1. Show that $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$ [Q.N. 2(b), 2057]

7.2 Differential Coefficients by definition (by first principle)

1. Find, from first principles, the derivative of xlnx. [Q.N.11(Or), 2072'C']
(Ans: $1 + \ell nx$)
2. Find from first principles the derivative of ln $\cos^{-1}x$. [Q.N.11(Or), 2072'D']
$\left(\text{Ans: } \frac{-1}{\sqrt{1-x^2}\cos^{-1}x}\right)$
3. Solve : $x\frac{dy}{dx} + y - 1 = 0$. 2[Q.N.4(a), 2072'E']
(Ans: $x(y-1) = e$)
4. Find from first principles the derivative of $\sin x^2$. [Q.N.11(Or), 2072'E']
(Ans: $2x\cos x^2$)
5. Find from first principle, the derivative of $\tan^{-1}x$. 6 [Q.N. 11(OR), Set 'C' 2071]
$\left(\text{Ans: } \frac{dy}{dx} = \frac{1}{1+x^2}\right)$
6. Find from first principle, the derivative of: $\ln(\sin\frac{x}{a})$.
$\left(\text{Ans: } \frac{d}{dx}\left(\log\sin\frac{x}{a}\right) = \frac{1}{a}\cot\frac{x}{a}\right)$ 6 [Q.N. 11(OR), Set 'D' 2071]
7. Find from first principles, the derivative of log (tanx). [Q.N. 11(OR), 2070 'C']
$\left(\text{Ans: } \frac{1}{\sin x.\cos x}\right)$
8. Find from first principles, that derivative of x^x. [Q.N. 11(OR), 2070 'D']
(Ans: $x^x(1 + \log x)$)
9. Find from first principles the derivative of $\log\cos^{-1}x$. [Q.N. 11(OR), Supp. 2069]
$\left(\text{Ans: } \frac{-1}{\cos^{-1}x\sqrt{1-x}}\right)$
10. Find from first principle, the derivative of sin(logx) [Q.N. 11(OR), Set 'A' 2069]
$\left(\text{Ans: } \frac{1}{x}\cos(\log x)\right)$
11. Find from first principle, the derivative of $e^{\sin x}$. [Q.N. 11(OR), Set 'B' 2069]
(Ans: $\cos x.\ e^{\sin x}$)
12. Find from first principle, the derivative of : $e^{\sin x}$ [Q.N. 10(b), 2068]
(Ans: $\cos x\ e^{\sin x}$)
13. Find from first principles, the derivative of : $\sin^{-1}x$. [Q.N. 10(b), 2067]
$\left(\text{Ans: } \frac{1}{\sqrt{1-x^2}}\right)$
14. Find the derivative of Sin (log x) from first principles. [Q.N. 10(b), 2066]
$\left(\text{Ans: } \frac{1}{x}\cos(\log x)\right)$
15. Find from first principles, the derivative of a^x. [Q.N.10(b), 2065]
(Ans: $a^x \log a$)
16. Find from first principles, the derivative of log sinx. [Q.N. 10(b), 2064]
(Ans: cotx)

17. Find from first principles, the derivative of *log tanx* [Q.N. 10(b), 2063]

$\left(Ans.: \frac{1}{\sin x \cdot \cos x}\right)$

18. Find from first principles, the derivative of $\tan^{-1}x$. [Q.N. 10(b), 2062]

$\left(Ans.: \frac{1}{1+x^2}\right)$

19. Find from first principles, the derivative of $e^{\sqrt{x}}$. [Q.N. 10(b), 2061]

$\left(Ans: \frac{1}{2\sqrt{x}} e^{\sqrt{x}}\right)$

20. Find from first principles, the derivative of $e^{\sin x}$ [Q.N. 10(b), 2060]

$\left(Ans: \cos x.e^{\sin x}\right)$

21. Find from first principles, the derivative of $e^{\tan x}$. [Q.N. 10(b), 2058]

$\left(Ans: \sec^2 x\, e^{\tan x}\right)$

7.3 Derivative of hyperbolic function

1. Find the derivative of: $\left(\text{Cosh}\frac{x}{a}\right)^{\log x}$ [Q.N. 2(b), 2068]

$\left[Ans: \left(\cosh\frac{x}{a}\right)^{\log x}\left(\frac{1}{a^2}\tanh\frac{x}{a}\log x + \frac{1}{x}\log\cosh\frac{x}{a}\right)\right]$

2. Find the derivative of $x^{\cosh\frac{x}{a}}$. [Q.N. 2(b), 2067]

$\left[Ans: x^{\cos h\frac{x}{a}}\left[\frac{\cos h\frac{x}{a}}{x} + \frac{\log x \sin h\frac{x}{a}}{a}\right]\right]$

3. Find the derivative of $2\tanh^{-1}\left(\tan\frac{1}{2}x\right)$ [Q.N. 2(b), 2066]

(Ans: secx)

4. Find the derivative of Arc tan Sin hx. [Q.N.2(b), 2065]

$\left(Ans: \frac{\cosh x}{1-\sinh^2 x}\right)$

5. Find the derivative of : $\left(\sin h\frac{x}{a}\right)^{x^2}$ [Q.N. 2(b), 2064]

$\left(Ans: \left(\sin\frac{x}{a}\right)^{x^2}\left[\frac{x^2}{a}\cot\frac{x}{a} + 2x\log\sinh\frac{x}{a}\right]\right)$

6. Find the derivative of $x^{Cosh^2\frac{x}{a}}$. [Q.N. 2(b), 2063]

$\left(Ans.: x^{\cosh^2\frac{x}{a}}\left(\frac{1}{x}.\cosh^2\frac{x}{a} + \frac{1}{a}.\log x.\sin\frac{2x}{a}\right)\right)$

7. Find the derivative of $\log(\sinh x/a)$ [Q.N. 2(b), 2062]

$\left(Ans.: \frac{1}{a}\coth\frac{x}{a}\right)$

8. Find the derivative of : $x^{\cos h\, x/a}$ [Q.N. 2(b), 2061]

$\left(\text{Ans: } x^{\cosh\frac{x}{a}}\left[\frac{\cosh x/a}{x}+\frac{\log x \sinh x/a}{a}\right]\right)$

9. Find the derivative of $2\tan^{-1}\left(\tanh\frac{x}{2}\right)$ [Q.N. 2(b), 2060]

(Ans: Sechx)

10. Find the derivative of $e^{\cosh^{-1}x}$. [Q.N. 2(a), 2059]

$\left(\textit{Ans: }\frac{1}{\sqrt{x^2-1}}e^{\cosh^{-1}x}\right)$

11. Find, from definition, the derivative of $e^{\tan x}$. [Q.N. 10(b), 2059]

$\left(\textit{Ans: } sec^2x e^{\tan x}\right)$

12. Find the derivative of x^{Coshx} [Q.N. 2(b), 2058]

$\left(\textit{Ans: } x^{\cosh x}\left(\frac{\cosh x}{x}+\log x \sinh x\right)\right)$

13. Find the derivative of $\left(\text{Sinh}\frac{x}{a}+\text{Cosh}\frac{x}{a}\right)^{nx}$ [Q.N. 10(b), 2057]

$\left(\text{Ans: } n\left(\sinh\frac{x}{a}+\cosh\frac{x}{a}\right)^{nx}\left[\frac{x}{a}+\log\left(\sinh\frac{x}{a}+\cosh\frac{x}{a}\right)\right]\right)$

7.4 Tangent and Normal

1. Find the points on the circle $x^2 + y^2 = 16$ at which the tangents are parallel to X-axis. 2[Q.N.3(a), 2072'E']
(Ans: (0, 4), (0, – 4))

2. Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangent is parallel to the x-axis. 2
(Ans: (0, 1), (2, – 3)) [Q.N. 3(a), Set 'C' 2071]

3. Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4)
(Ans: 4x – y – 4 = 0) 2 [Q.N. 3(a), 2070 'D']

4. At what angle does the curve $y(1+x) = x$ cut the x-axis ? [Q.N. 5(c), 2068]

$\left(\textit{Ans: }\frac{\pi}{4}\right)$

5. Find the angle of intersection between the curves $y = x^2$ and $6y = 7 - x^3$ at (1, 1). [Q.N. 2(c), 2067]

$\left(\textit{Ans: } \tan^{-1}\left(\pm\frac{3}{2}\right)\right)$

6. Find the angle of intersection of the curves $y^2 = x^3$ and $y = 2x$ at the point (0, 0).

$\left(\textit{Ans: } \tan^{-1}(\pm 2) \text{ at (0, 0) } \tan^{-1}\left(\pm\frac{1}{7}\right) \text{ at (4, 8)}\right)$ [Q.N. 2(c), 2066]

7. Find where the tangent is parallel to the x-axis for the curve $y = x^3 - 3x^2 - 9x + 15$. [Q.N.2(c), 2065]
(Ans: (–1, 20) and (3, –12))

8. Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangents are parallel to x-axis.
(Ans: (0, 1), (2, –3)) [Q.N. 2(c), 2064]

9. Find the angle of intersection of the curves [Q.N. 2(c), 2062]
$4y = x^2 + 12$ and $y^2 = 8x$ at (2,4)
(Ans.: 0°)

10. Find the slope and inclination with the x-axis of the tangent of $y = -3x - x^4$ at $x = -1$.

$\left(\textit{Ans: 1 and } \frac{\pi}{4}\right)$ [Q.N. 2(c), 2061]

11. Find the points on the curve $4y = x^4 - 8x^2$ where the tangents are paralleled to the x-axis. [Q.N. 5(c), 2060]
(Ans: (0, 0), (–2, –4) and (2, –4)

12. Find the slope and inclination with x -axis of the tangent of: $x^2 + y^2 = 36$ at (0,6).
(Ans: 0 and 0°) [Q.N. 5(c), 2058]

13. Find the slope and inclination with X -axis of the tangent of the curve $2y = 2 - x^2$ at x =1. [Q.N. 5(c), 2057]
$\left(\text{Ans: } -1, \frac{3\pi}{4}\right)$

7.5 L Hospital's rule, Roll's Theorem & Mean Value Theorem

1. Evaluate, using L' Hospitals rule: $\lim_{x \to 0} \frac{\tan ax}{\tan bx}$. 2[Q.N.3(a), 2072'C']
$\left(\text{Ans: } \frac{a}{b}\right)$

2. State Rolle's theorem. Verify Rolle's theorem for the functions $f(x) = 2x^3 - 3x + 1$ in $\left[\frac{1}{2}, 1\right]$. 6[Q.N.11, 2072'C']

3. Using L' Hospitals rule, evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$. 2[Q.N.3(b), 2072'D']
$\left(\text{Ans: } \frac{1}{2}\right)$

4. State Mean Value theorem. Verify the mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$, $x \in [2, 4]$. 6[Q.N.11, 2072'D']
(Ans: $c = \sqrt{6}$)

5. State Rolle's theorem. What is the geometrical interpretation of Rolle's theorem. Verify Rolle's theorem for the function $f(x) = \sqrt{1 - x^2}$, $x \in [-1, 1]$. 6[Q.N.11, 2072'E']
(Ans: c = 0)

6. State mean value theorem. Interpret it geometrically. Verify the mean value theorem for the function $f(x) = (x - 1)(x - 2)(x - 3)$ in [1, 4]. 6 [Q.N. 11, Set 'C' 2071]

7. Using L Hospital's rule, evaluate:
$\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$. 2 [Q.N. 3(a), Set 'D' 2071]
[Ans: 1]

8. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = x(x-3)^2$ for $x \in [0, 3]$. 6 [Q.N. 11, Set 'D' 2071]

9. Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$ 2 [Q.N. 3(a), 2070 'C']
[Ans: 2]

10. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = x(x-3)^2$ for $x \in [0, 3]$. 6 [Q.N. 11, 2070 'C']
[Ans: C = 1]

11. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x(x - 1)^2$ in [0, 2]. 6 [Q.N. 11, 2070 'D']
$\left(\text{Ans: } c = \frac{4}{3}\right)$

12. Evaluate, using L'Hospital rule $\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$ [Q.N. 3(a), Supp. 2069]2
[Ans: 1]

13. Using mean value theorem, find a point on the parabola $y = (x-3)^2$ where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

$\left(\text{Ans: } \frac{7}{2}, \frac{1}{4}\right)$ 6 [Q.N. 11, Supp. 2069]

14. Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$ [Q.N. 3(a), Set 'A' 2069]

$\left(\text{Ans: } \frac{1}{2}\right)$

15. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x^3 + x^2 - 6x$ in $[-1, 4]$. 6 [Q.N. 11, Set 'A' 2069]

16. Using L Hospital's rule, evaluate:

$\lim_{x \to 0} \frac{x - \sin x}{x^3}$ [Q.N. 3(a), Set 'B' 2069]

$\left(\text{Ans: } \frac{1}{6}\right)$

17. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[1,4]$. [Q.N. 11, Set 'B' 2069]

Unit 8: Antiderivatives

1. Evaluate : $\int \frac{dx}{1 - 2\cos x}$ 2[Q.N.3(b), 2072'C']

$\left(\text{Ans: } \frac{1}{\sqrt{3}} \log \frac{\sqrt{3}\tan\frac{x}{2} - 1}{\sqrt{3}\tan\frac{x}{2} + 1} + C\right)$

2. Evaluate : $\int \frac{dx}{(x-2)^2 (x-3)^3}$ 4[Q.N.7(a), 2072'C']

$\left(\text{Ans: } -\frac{1}{2}\left(\frac{x-2}{x-3}\right)^2 + 3\left(\frac{x-2}{x-3}\right) - \log\frac{x-2}{x-3} - \frac{x-3}{x-2} + C\right)$

3. Compute the integral $\int \frac{\coth x\, dx}{\sinh x - 9\,\text{cosech} x}$ 2[Q.N.3(a), 2072'D']

$\left(\text{Ans: } \frac{1}{6} \log \frac{\sinh x - 3}{\cosh x + 3} + c\right)$

4. Evaluate : $\int \frac{dx}{(x-1)^2 (x-2)^3}$ 4[Q.N.7(a), 2072'D']

$\left(\text{Ans: } -\frac{1}{2}\left(\frac{x-1}{x-2}\right)^2 + 3\left(\frac{x-1}{x-2}\right) - 3\log\left(\frac{x-1}{x-2}\right) - \frac{x-2}{x-1} + c\right)$

5. Evaluate: $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}} (\beta > \alpha)$ 2[Q.N.3(b), 2072'E']

(Ans: $2\log(\sqrt{x-\alpha} + \sqrt{x-\beta}) + c$)

6. Evaluate : $\int \frac{dx}{2+3\cos x}$ 4[Q.N.7(a), 2072'E']

$$\left(\text{Ans: } \frac{1}{\sqrt{5}} \frac{\log \sqrt{5} + \tan \frac{x}{2}}{\sqrt{5} - \tan \frac{x}{2}} + C\right)$$

7. Evaluate : $\int \frac{6x+1}{x^2+9} dx$. 2 [Q.N. 3(b), Set 'C' 2071]

$$\left(\text{Ans: } 3 \log (x^2 + 9) + \frac{1}{3} \tan^{-1} \frac{x}{3} + C\right)$$

8. Evaluate: $\int \frac{dx}{1-2\cos x}$. 4 [Q.N. 7(a), Set 'C' 2071]

$$\left(\text{Ans: } \frac{1}{\sqrt{3}} \log \left(\frac{\sqrt{3} \tan \frac{x}{2} - 1}{\sqrt{3} \tan \frac{x}{2} + 1}\right) + C\right)$$

9. Evaluate : $\int \frac{dx}{\sqrt{2ax+x^2}}$. 2 [Q.N. 3(b), Set 'D' 2071]

[Ans: $\log (x + a + \sqrt{x^2 + 2ax}) + C$

10. Evaluate: $\int \frac{dx}{1-3\sin x}$ 4 [Q.N. 7(a), Set 'D' 2071]

$$\left(\text{Ans: } \left(\frac{1}{2\sqrt{2}} \log \frac{\tan \frac{x}{2} - 3 - 2\sqrt{2}}{\tan \frac{x}{2} - 3 + 2\sqrt{2}} + C\right)\right)$$

11. Evaluate : $\int \frac{2x+3}{4x^2+1} dx$. 2 [Q.N. 3(b), 2070 'C']

$$\left(\text{Ans: } \frac{1}{4} \log (4x^2 + 1) + \frac{3}{2} \tan^{-1} 2x + C\right)$$

12. Evaluate: $\int \frac{dx}{2+\cos x}$ 4 [Q.N. 7(a), 2070 'C']

$$\left(\text{Ans: } \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{\tan^{-1} \frac{x}{2}}{\sqrt{3}}\right) + C\right)$$

13. Evaluate : $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$, $(\beta > \alpha)$ 2 [Q.N. 3(b), 2070 'D']

(Ans: $2\log (\sqrt{x-\alpha} + \sqrt{x-\beta}) + C$)

14. Evaluate: $\int \frac{dx}{1+2\sin x}$ 4 [Q.N. 7(a), 2070 'D']

$$\left(\text{Ans: } \frac{1}{\sqrt{3}} \log \frac{\tan \frac{x}{2} + 2 - \sqrt{3}}{\tan \frac{x}{2} + 2 + \sqrt{3}} + C\right)$$

15. Evaluate : $\int \frac{dx}{1+\sin x+\cos x}$. 2 [Q.N. 3(b), Supp. 2069]

$\left(\text{Ans: } \log\left(1+\tan\frac{x}{2}\right)+c\right)$

16. Evaluate: $\int \frac{x^2}{x^4-2x^2-15}\,dx$. 4 [Q.N. 7(a), Supp. 2069]

$\left(\text{Ans; } \frac{\sqrt{3}}{8}\tan^{-1}\frac{x}{3}+\frac{\sqrt{5}}{16}\log\frac{x-\sqrt{5}}{x+\sqrt{5}}+C\right)$

17. Evaluate : $\int \frac{dx}{\sqrt{2ax-x^2}}$. [Q.N. 3(b), Set 'A' 2069]

$\left(\text{Ans: } \sin^{-1}\frac{x-a}{a}+c\right)$

18. Evaluate: $\int \frac{dx}{3\sin x-4\cos x}$. [Q.N. 7(a), Set 'A' 2069]

$\left(\text{Ans: } \frac{1}{5}\frac{\tan\frac{x}{2}-\frac{1}{2}}{\tan\frac{x}{2}+2}+c\right)$

19. Evaluate : $\int \frac{dx}{e^x+e^{-x}}$. [Q.N. 3(b), Set 'B' 2069]

(Ans: $\tan^{-1} e^x + c$)

20. Evaluate: $\int \frac{dx}{2+3\cos x}$. [Q.N. 7(a), Set 'B' 2069]

$\left(\text{Ans: } \frac{1}{\sqrt{5}}\log\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}+C\right)$

21. Evaluate: $\int \frac{dx}{\sqrt{2ax-x^2}}$ [Q.N. 3(a), 2068]

$\left(\text{Ans: } \sin^{-1}\frac{x-a}{a}+C\right)$

22. Evaluate: $\int \frac{dx}{1+\sin x+\cos x}$ [Q.N. 11(b), 2068]

$\left(\text{Ans: } \log\left(1+\tan\frac{x}{2}\right)+C\right)$

23. Evaluate : $\int \frac{1}{x^2} e^{\frac{-1}{x}}\,dx$ [Q.N. 3(a), 2067]

(Ans: $e^{-\frac{1}{x}} + C$)

24. Evaluate : $\int \frac{dx}{1+2\sin x}$ [Q.N. 11(b), 2067]

$\left(\text{Ans: } \frac{1}{\sqrt{3}}\log\frac{\tan\frac{x}{2}+2-\sqrt{3}}{\tan\frac{x}{2}+2+\sqrt{3}}+C\right)$

25. Integrate: $\int \frac{dx}{\sqrt{2ax + x^2}}$ [Q.N. 3(a), 2066]

(Ans: $\log(x + a) + \sqrt{x^2 + 2ax} + c$)

26. Integrate: $\int \frac{x^2}{(x + 2)(x + 3)^2} dx$ [Q.N. 11(b), 2066]

$\left(\text{Ans: } 4\log(x + 2) - 3\log(x + 3) + \frac{9}{x + 3} + c\right)$

27. Evaluate : $\int \frac{dx}{e^x + e^{-x}}$ [Q.N.3(a), 2065]

(Ans: $\tan^{-1}(e^x) + c$)

28. Integrate : $\int \frac{dx}{3 + 4\cos hx}$ [Q.N.11(b), 2065]

$\left(\text{Ans: } \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{\tan h\frac{x}{2}}{\sqrt{7}}\right) + c\right)$

29. Evaluate : $\int \frac{dx}{x^2 - 16}$ [Q.N. 3(a), 2064]

$\left(\text{Ans: } \frac{1}{8}\log\frac{x - 4}{x + 4} + C\right)$

30. Integrate : $\int \frac{\cos x - \sin x}{\sqrt{\sin 2x}} dx$ [Q.N. 11(b), 2064]

$\left(\text{Ans: } \log\left\{(\sin x + \cos x) + \sqrt{\sin 2x}\right\} + c\right)$

31. Integrate : $\int \frac{dx}{1 + \sin x + \cos x}$ [Q.N. 11(b), 2063]

$\left(\text{Ans.: } \log\left(1 + \tan\frac{x}{2}\right) + C\right)$

32. Integrate : $\int \frac{dx}{\sqrt{2ax - x^2}}$ [Q.N. 3(a), 2062]

$\left(\text{Ans.: } \sin^{-1}\frac{x - a}{a} + C\right)$

33. Integrate : $\int \frac{dx}{\sin x + \cos x}$ [Q.N. 11(b), 2062]

$\left(\text{Ans.: } \frac{1}{\sqrt{2}}\log\left[\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)\right] + C\right)$

34. Evaluate : $\int \sqrt{\frac{1 + x}{1 - x}} dx$ [Q.N. 3(a), 2061]

$\left(\text{Ans: } -\sqrt{1 - x^2} + \sin^{-1} x + c\right)$

35. Find the value of : $\int \frac{dx}{3\sin x - 4\cos x}$ [Q.N. 11(b), 2061]

$$\left(\text{Ans: } \frac{1}{5}\log\frac{\tan\frac{x}{2}-\frac{1}{2}}{\tan\frac{x}{2}+2}+C\right)$$

36. Evaluate : $\int \frac{dx}{\sqrt{a^2-x^2}}$ [Q.N. 3(a), 2060]

$$\left(\text{Ans: } \sin^{-1}\frac{x}{a}+C\right)$$

37. Find the value of :

$$\int \frac{\sin x \cdot \cos x}{(\sin x+\cos x)^2}dx$$ [Q.N. 11(b), 2060]

$$\left(\text{Ans: } -\frac{1}{2}\frac{1}{(\tan x+1)}+C\right)$$

38. $\int \frac{dx}{a+b\cos x}$ a<b . [Q.N. 11(b), 2059]

$$\left(\text{Ans: } \frac{1}{\sqrt{b^2-a^2}}\log\left(\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}}\right)+C\right)$$

39. Prove that: $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+c.$ [Q.N. 3(a), 2058]

40. Integrate: $\int \sqrt{\frac{1+x}{1-x}}dx.$ [Q.N. 11(b), 2058]

$$\left(\text{Ans: } -\sqrt{1-x^2}+\sin^{-1}x+c\right)$$

41. Prove: $\int \text{Cosec}x\,dx = \log\left|\tan\frac{x}{2}\right|+c$ [Q.N. 3(a), 2057]

42. Integrate: $\int \frac{dx}{a+b\cos x}$ when a> b. [Q.N. 11(b), 2057]

$$\left(\text{Ans: } \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)+C\right)$$

Unit 9: Differential Equations and their Applications

1. Solve : $\frac{dy}{dx}+\frac{y}{x}=1.$ 2[Q.N.4(a), 2072'C']

$$\left(\text{Ans: } xy=\frac{x^2}{2}+C\right)$$

2. Solve : $\frac{dy}{dx} = y\tan x - 2\sin x$ 4[Q.N.7(b), 2072'C']

$\left(\text{Ans: } y\cos x = \frac{\cos 2x}{2} + C\right)$

3. Solve : $xy\frac{dy}{dx} - y^2 = x^2$ 4[Q.N.7(b)(Or), 2072'C']

(Ans: $y^2 = 2x^2(\log x + C)$)

4. Solve : $\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2y} = 0$. 2[Q.N.4(a), 2072'D']

(Ans: $\tan y - y + x = c$)

5. Reduce the equation $\frac{dy}{dx} + \frac{y}{x} = y^2$ in liner form hence solve it. 4[Q.N.7(b), 2072'D']

$\left(\text{Ans: } \frac{1}{xy} = -\log x + c\right)$

6. Solve : $\frac{dy}{dx} = \frac{y+1}{x+y+1}$ [Q.N.7(b)(Or), 2072'D']

(Ans: $y + 1 = ce^{\frac{x}{y+1}}$)

7. Solve: $\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$. 4[Q.N.7(b), 2072'E']

$\left(\text{Ans: } \cot\left(\frac{y}{x}\right) = \log x + c\right)$

8. Solve :$\sin x\frac{dy}{dx} + \cos xy = x\sin x$. [Q.N.7(b)(Or), 2072'E']

(Ans: $y\sin x = -x\cos x + \sin x + c$)

9. Solve: $e^{x-y}.dx + e^{y-x}dy = 0$. 2 [Q.N. 4(a), Set 'C' 2071]

(Ans: $e^{2x} + e^{2y} = C$)

10. Solve: $\frac{dy}{dx} = \frac{x^2+y^2}{2x^2}$. 4 [Q.N. 7(b), Set 'C' 2071]

(Ans: $2x = (x - y)\log(cx)$)

11. Solve: $\sin x\frac{dy}{dx} + (\cos x).y = \sin x.\cos x$. 4 [Q.N. 7(b)(OR), Set 'C' 2071]

$\left(\text{Ans: } y\sin x + \frac{1}{4}\cos 2x = c\right)$

12. Solve: $\frac{dy}{dx} + 4x = 2e^{2x}$. 2 [Q.N. 4(a), Set 'D' 2071]

[Ans: $y = e^{2x} - 2x^2 + C$

13. Solve: $xy\frac{dy}{dx} = x^2 + y^2$. 4 [Q.N. 7(b), Set 'D' 2071]

[Ans: $y^2 = 2x^2(\log x + C)$

14. Solve: $\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{1}{(1+x^2)^2}$. 4 [Q.N. 7(b)(OR), Set 'D' 2071]

[Ans: $y(1 + x^2) = \tan^{-1}x + C$)

15. Solve: $\frac{dy}{dx} = \frac{e^x + 1}{y}$ 2 [Q.N. 4(a), 2070 'C']

[Ans: $y^2 = 2e^x + 2x + C$]

16. Solve: $\cos^2 x\frac{dy}{dx} + y = 1$. [Q.N. 7(b), 2070 'C']

[Ans: $y = 1 + Ce^{-\tan x}$]

17. Solve: $\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$. [Q.N. 7(b)(OR), 2070 'C']

$\left(\text{Ans: } \cot\frac{y}{x} = \log x + C\right)$

18. Solve: $x^2dy - y^2dx = 0$ 2 [Q.N. 4(a), 2070 'D']

(Ans: $y - x = Axy$)

19. Solve: $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$. 4 [Q.N. 7(b), 2070 'D']

$\left(\text{Ans: } (1 + x^2)\, y = \frac{4}{3}x^3 + C\right)$

20. Solve: $(x^2 + y^2)dy = xydx$ [Q.N. 7(b)(OR), 2070 'D']

(Ans: $x^2 = 2y^2 \log(yc)$)

21. Solve: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$. 2 [Q.N. 4(a), Supp. 2069]

[Ans: $y - x = C(1 + xy)$]

22. Solve: $\tan x\frac{dy}{dx} + y = \sec x$ 4 [Q.N. 7(b), Supp. 2069]

[Ans: $y \sin x = x + C$]

23. Solve: $\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$ [Q.N. 7(b)(OR), Supp. 2069]

[Ans: $x^2 + y^2 - 2xy + 10y - 2x = C$]

24. Solve: $\frac{dy}{dx} = e^{x-y} + x^3 \cdot e^{-y}$ [Q.N. 4(a), Set 'A' 2069]

$\left(\textit{Ans: } e^y = e^x + \frac{x^4}{4} + c\right)$

25. Solve: $\tan x\frac{dy}{dx} + y = \sec x$ 4 [Q.N. 7(b), Set 'A' 2069]

(Ans: $y \sin x = x + c$)

26. Solve: $xy\frac{dy}{dx} = x^2 + y^2$ [Q.N. 7(b)(OR), Set 'A' 2069]

(Ans: $y^2 = 2x^2(\log x + c)$)

27. Solve: $e^{x-y}\,dx + e^{y-x}.\,dy = 0$ [Q.N. 4(a), Set 'B' 2069]

(Ans: $e^{2x} + e^{2y} = C$)

28. Solve: $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ [Q.N. 7(b), Set 'B' 2069]

$\left(\textit{Ans: } (1 + x^2)\, y = \frac{4}{3}x^3 + C\right)$

29. Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [Q.N. 7(b)(OR), Set 'B' 2069]

$\left(\textit{Ans: } \sin\left(\frac{y}{x}\right) = Cx\right)$

30. Solve: $x^2dy - y^2dx = 0$ [Q.N. 6(c), 2068]

(Ans: $y - x = c\,y$, c is a constant.)

31. Solve: $\sin x\frac{dy}{dx}\cos x.y = x \sin x$ [Q.N. 11(b)(Or), 2068]

(Ans: $y + x\cot x - c\,\text{cosec } x = 1$; c is constant.)

32. Solve : $xdy + (x+y)\,dx = 0$ [Q.N. 6(c), 2067]

(Ans: $2xy + x^2 = c^2$)

33. Solve : $(1-x^2)\frac{dy}{dx} = 1 + xy$ [Q.N. 11(b) (Or)), 2067]

(Ans: $\sqrt{1-x^2}\ y = \sin^{-1} x + C$)

34. Solve the differential equation $(x + 2y - 3)\,dy - (2x - y + 1)\,dx = 0$. [Q.N. 6(c), 2066]

(Ans: $xy + y^2 - x^2 - 3y - x = c$)

35. Solve the differential equation: $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ [Q.N. 11(b)Or, 2066]

$\left(Ans: y = \frac{1}{2}e^{\tan^{-1}x} + ce^{-\tan^{-1}x}\right)$

36. Solve: $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$. [Q.N.6(c), 2065]

(Ans: $2x^3 + 2y^3 + 3x^2y^2 = c$)

37. Solve: $\frac{dy}{dx} + y \cot x = x$. [Q.N.11(b, or), 2065]

(Ans: $y \sin x = -x \cos x + \sin x + c$)

38. Solve : $xdy + (x + y)\,dx = 0$ [Q.N. 6(c), 2064]

(Ans: $y = x \log (cx)$)

39. Solve : $2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$ [Q.N. 11(b)Or, 2064]

$\left(Ans: (y-x)^2 = cxy^2\right)$

40. Solve : $e^{x-y}\,dx + e^{y-x}\,dy = 0$ [Q.N. 6(c), 2063]

(Ans.: $e^{2x} + e^{2y} = C$)

41. Solve : $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$. [Q.N. 11(b) Or, 2063]

$\left(Ans.: \sin\frac{y}{x} = cx\right)$

42. Solve : $x^2dy - y^2dx = 0$ [Q.N. 6(c), 2062]

(Ans.: $y - x = Axy$)

43. Solve : $\frac{dy}{dx} + \frac{1}{x} \cdot y = x^2$ [Q.N. 11(b)Or, 2062]

$\left(Ans.: xy = \frac{x^4}{4} + C\right)$

44. Solve : $x^2\,dy - y^2\,dx = 0$ [Q.N. 6(c), 2061]

(Ans: $y - x = Axy$)

45. Solve: $(x^2 - y^2)\frac{dy}{dx} = xy$ [Q.N. 11(b)Or, 2061]

(Ans: $2y^2\ logyC + x^2 = 0$)

46. Solve : $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$ [Q.N. 6(c), 2060]

(Ans: $y = \tan^{-1} x + C$)

47. Solve: $\tan x\frac{dy}{dx} + y = \sec x$ [Q.N. 11(b)Or, 2060]

(Ans: $y \sin x = x + C$)

48. Solve: $x\,dy - y\,dx = 0$. [Q.N. 2(b), 2059]

(Ans: $y = cx$)

49. Solve: $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ [Q.N. 11(b) Or, 2059]

$\left(Ans: y^2 + x^2 = cx\right)$

50. Solve: $\frac{dy}{dx} = \frac{x^3+1}{y^3+1}$ [Q.N. 6(c), 2058]

$\left(Ans: \frac{y^4}{4} + y = \frac{x^4}{4} + x + c\right)$

51. Solve : $\cos^2 x \frac{dy}{dx} + y = 1$. [Q.N. 11(b)Or, 2058]

$\left(Ans: y = 1 + ce^{-\tan x}\right)$

52. Solve: $\frac{dy}{dx} = \frac{x^2+x+1}{y^2+y+1}$ [Q.N. 6(c), 2057]

$\left(Ans: \frac{y^3}{3} + \frac{y^2}{2} + y = \frac{x^3}{3} + \frac{x^2}{2} + x + A\right)$

53. Solve: $\tan x \frac{dy}{dx} + y = \sec x$. [Q.N. 11(b)Or, 2057]

(Ans: ysinx = x + c)

Unit 10: Dispersion, Correlation and Regression

10.1 Measures of dispersions

1. The information about the daily temperature of two cities X and y are as follows:

	X	Y
Average temp. (°F)	84	92
Variance of temp.	16	25

Determine which city has greater consistency in climate. 2[Q.N.4(b); 2072'C']

(Ans: City X)

2. In the distribution of data 20, 25, 30, 36, 32, 43 ; find standard deviation.
(Ans: 7.39) 2[Q.N.4(b), 2072'D']

3. For a group of 50 items; circle $\Sigma x^2 = 600$, $\Sigma x = 150$ and $m_0 = 1.75$, find the Pearsonian coefficient of skewness. 2[Q.N.4(b), 2072'E']
(Ans: 0.72)

4. Calculate the coefficient of Skewness based on mean, mode and standard deviation from the following data: 4 [Q.N. 8(a), Set 'C' 2071]

Wages (in Rs.)	100	110	120	130	140
No. of persons	2	6	10	8	4

(Ans: 0.18)

5. If n = 10, $\Sigma x = 120$, $\Sigma x^2 = 1530$, find the standard deviation and the coefficients of variation. 2 [Q.N. 4(b), Set 'D' 2071]
[Ans: r = 3, C.V. = 25%]

6. If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, N = 10 and $M_0 = 12.45$ find the skewness based on mean, mode and standard deviation. 4[Q.N. 8(a), 2070 'C']
[Ans: –0.22]

7. Consider the following distribution.

	Distribution A	Distribution B
Arithmetic mean:	100	90
Median:	90	80
Standard deviation:	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skewness? 4[Q.N. 8(a), 2070 'D']
(Ans: Yes, same)

8. The information about the daily temperature of two cities A and B are as follows:

	City A	City B
Average Temperature (F)	84	92
Variance of Temperature	16	25.

Determine which city has greater consistency in climate.
[Ans: A] 2 **[Q.N. 4(b), Supp. 2069]**

9. Determine the standard deviation and the coefficient of variation from the following distribution, 4 **[Q.N. 8(a), Set 'A' 2069]**

Profit (in Rs.)	0-10	10-20	20-30	30-40	40-50
No. of shops	8	13	16	8	5

(Ans: Rs. 11.88, 52.11%)

10. If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, $N = 10$ and $M_0 = 12.45$, find the skewness based on mean, mode and standard deviation. **[Q.N. 8(a), Set 'B' 2069]**
(Ans: – 0.22)

11. Weights of a group of individuals are given below. Find out the mean and the standard deviation. **[Q.N. 12(a), 2068]**

Weight (in kg.)	0-10	10-20	20-30	30-40	40-50
Frquency	12	33	30	15	10

(Ans: 27, 11.45)

12. Consider the following distribution:

	distribution A	distribution B
Arithmetic mean	100	90
Medium	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skewness ? **[Q.N. 12(b), 2068]**
(Ans: Yes, same)

13. In a frequency distribution of a set of data C.V. = 5%, $\sigma = 2$ and Karl Pearson coefficient of skewness = 0.5; find the mean of the data. **[Q.N. 4(b), 2067]**
(Ans: 40)

14. Define standard deviation. Also prove that the root mean square deviation is not less than the standard deviation. **[Q.N. 12(b), 2067]**

15. Find the standard deviation from the following data:

x:	10	11	12	13	14
f:	3	12	18	12	2

(Ans: 0.967) **[Q.N. 4(b), 2066]**

16. Calculate the coefficient of skewness from the following frequency distribution: **[Q.N.12(b), 2066]**

Investment	10-20	20-30	30-40	40-50	50-60	60-70	70-80
	12	18	20	15	10	3	2

(Ans: 0.227)

17. Calculate Karl Pearson's coefficient of skewness of the data: **[Q.N.12(a), 2065]**

Marks	above 0	above 10	above 20	above 30	above 40
Frequency	150	140	100	80	80

(Ans: 0.89)

18. The information about the wages distribution of the firms A and B are given below :

	Firm A	Firm B
No. of workers	500	600
Average monthly wages	Rs. 586	Rs. 575
Variable of wages distribution	81	100

In which firm is the wages distribution uniform ? **[Q.N. 4(b), 2064]**
(Ans: firm A)

19. For a group of 10 items, $\Sigma x = 452$, $\Sigma x^2 = 24,270$ and mode = 43.7, find the Pearson's coefficient of skewness. [Q.N. 12(a), 2064]
(Ans: 0.076)

20. Consider the following distribution.

	Distribution a	Distribution B
Arithmetic mean:	100	90
Median:	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the skewness ?
(Ans.: Same) [Q.N. 4(b), 2063]

21. Find the standard deviation of the following data :
10, 15, 20, 25, 30, 35, 40 [Q.N. 4(b), 2062]
(Ans.: 10)

22. Following are the marks obtained by two students A and B in 10 tests of 100 marks each.

Test:	1	2	3	4	5	6	7	8	9	10
Marks of A :	44	80	76	48	52	72	68	56	60	54
Marks of B :	48	75	54	60	63	69	72	51	57	66

Who is intelligent ? [Q.N. 12(a), 2062]
(Ans.: B)

23. Find the mean deviation from mean of the following data :
6, 8, 10, 13 and 5. [Q.N. 4(b), 2061]
(Ans: 2.72)

24. Find out the mean and Standard Deviation from the following data :

Variable:	5-10	10-15	15-20	20-25	25-30	30-35
Frequency:	2	9	29	54	11	5

(Ans: 23.95 and 4.87) [Q.N. 12(a), 2061]

25. The median, mode and coefficient of skewness for a certain distribution are respectively 17.4, 15.3 and 0.35. Calculate mean and C.V. [Q.N. 12(b), 2061]
(Ans: 18.45 and 48.78%)

26. Find the mean deviation from median of the numbers 5, 7, 10, 12 and 6.
(Ans: 1.57) [Q.N. 4(b), 2060]

27. Find the mean and S.D. from the following table wages (Rs.) :

Wages:	10–20	10–30	10–40	10–50	10–60
No. of workers :	15	33	63	83	100

(Ans: Mean = 35.06 and S.d. = 12.45) [Q.N. 12(a), 2060]

28. Prove that in a discrete distribution the standard deviation is not less than the mean deviation from the mean. [Q.N. 12(b), 2060]

29. The coefficient of variation and mean of a certain frequency distribution are 50.2% and 22.8 respectively. Find the s.d. [Q.N. 4(c), 2059]
(Ans: 11.4456)

30. Find the standard deviation of the following data:
100, 150, 200, 250, 300. [Q.N. 4(b), 2058]
(Ans: 70.7)

31. Write the expressions for the first four central moment and for β_2. Hence show that for a discrete distribution, $\beta_2 \geq 1$. [Q.N. 12(b), 2057]

$$\left(\textit{Ans: } \mu_1 = 0, \mu_2 = \frac{\Sigma x^2}{N}, \mu_3 = \frac{\Sigma x^3}{N}, \mu_4 = \frac{\Sigma x^4}{N}, \beta_2 = \frac{\mu_4}{\mu_2} \right)$$

10.2 Correlation

1. Define correlation. Find Karl Pearson's coefficient of correlation of the marks of the following distribution. **4[Q.N.8(a), 2072'D']**

X	20	30	40	50	60
Y	50	46	30	24	8

(Ans: – 0.937)

2. If n = 15, σ_X = 3.2, σ_Y = 3.4 and $\Sigma(X - \bar{X})(Y - \bar{Y})$ = 122, find the correlation coefficient between the two variables. **2 [Q.N. 4(b), Set 'C' 2071]**
(Ans: 0.75)

3. Calculate Karl Pearson's correlation coefficient between the two variables height (in cms) and weight (in kg) from the data gives below: **4 [Q.N. 8(a), Set 'D' 2071]**
[Ans: 0.51]

Height	160	162	165	161	163
Weight	63	62	64	60	61

4. If $\Sigma(X - \bar{X})^2 = 40$, $\Sigma(Y - \bar{Y})^2 = 63$ and $\Sigma(X - \bar{X})(Y - \bar{Y}) = 35$, find the correlation coefficient between the two variables. **2 [Q.N. 4(b), 2070 'C']**
[Ans: 0.697]

5. If n = 10, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. **2 [Q.N. 4(b), 2070 'D']**
(Ans: 0.59)

6. Find Karl Pearsons' coefficient of correlation of the marks of the following two group of students **4 [Q.N. 8(a), Supp. 2069]**
[Ans: –0.937]

X	20	30	40	50	60
Y	50	46	30	24	8

7. Calculate the correlation coefficient between two variables from the following data: $\Sigma x^2 = 114$, $\Sigma y^2 = 422$, and $\Sigma xy = 174$ **[Q.N. 4(b), 2068]**
[Ans: 0.793)

8. If the covariance between the variable x and y is 18 and the variances of x and y are 16 and 81 respectively, find the coefficient of correlation between them.
(Ans: 0.5) **[Q.N.4(b), 2065]**

9. From the following table, calculate the coefficient of correlation by Karl Pearson's method. **[Q.N. 12(b), 2063]**

X :	6	2	10	4	8
Y :	9	11	–	8	7

Arithmetic means of X and Y series are 6 and 8 respectively.
(Ans.: – 0.92)

10. Prove that the correlation coefficient between two variables lies between –1 and +1. **[Q.N. 12(b), 2062]**

11. Calculate Karl Pearson's coefficient of correlation from the following data:

x:	12	9	8	10	11	13	7
y:	14	8	6	9	11	12	3

(Ans: 0.95) **[Q.N. 12(b), 2059]**

12. Prove that the coefficient of correlation lies between-1 and 1. **[Q.N. 12(b), 2058]**

13. Calculate r_{xy} if $\Sigma x^2 = 114$; $\Sigma y^2 = 442$; $\Sigma xy = 174$. **[Q.N. 4(b), 2057]**
(Ans: 0.793)

10.3 Regression

1. Define regression and lines of regression. Find the correlation coefficients between the two variables when $b_{xy} = 1.8$ and $b_{yx} = 0.35$. 4[Q.N.8(a), 2072'C']
   (Ans: 0.79)
2. The regression coefficients of x on y and y on x are 0.84 and 0.32 respectively. if the arithmetic means of x and y series are 42 and 26 respectively, find two equations of lines of regression. 4[Q.N.8(a), 2072'E']
   (Ans: y = 0.32x + 12.56, x + 0.84y + 20.16)
3. Find the regression equation of y on x when:
   $\Sigma x = 15, \Sigma y = 25, \Sigma x^2 = 55, \Sigma y^2 = 140, \Sigma xy = 78, n = 5.$
   (Ans: y = 0.3x + 4.1) [Q.N. 4(b), Set 'A' 2069]
4. The regression coefficient of y on x is 0.32. If the arithmetic means of x and y series are 42 and 36 respectively, find the regression equation of y on x.
   (Ans: 0.32x – y – 6 = 0) [Q.N. 4(b), Set 'B' 2069]

Unit 11: Probability

11.1 Probability

1. In rolling a pair of dice, determine the probability of obtaining a sum of 10.
   $\left(\text{Ans: } \frac{1}{12}\right)$ 2[Q.N.4(c), 2072'C']
2. In a draw of a card from well shuffled deck of 52 cards what is the probability that it is a king or a queen? 2[Q.N.4(c), 2072'D']
   $\left(\text{Ans: } \frac{2}{13}\right)$
3. Two dice are rolled once. What is the probability of getting a total of 8 or 7 ?
   $\left(\text{Ans: } \frac{11}{36}\right)$ 2[Q.N.4(c), 2072'E']
4. The chance that A can solve the problem is $\frac{3}{5}$ and the chance that B can solve the problem is $\frac{2}{3}$. Find the probability that the problem is solved. 2[Q.N. 4(c), Set 'C' 2071]
   $\left(\text{Ans: } \frac{13}{15}\right)$
5. Two coins are tossed simultaneously. Find the sample space. Find the probability that both are heads. 2 [Q.N. 4(c), Set 'D' 2071]
   [Ans: S ={HH, HT, TH, TT}, $P(HH) = \frac{1}{4}$]
6. A class consists of 60 boys and 40 girls. If two students are chosen at random, what is the probability that one is boy and one girl? 2 [Q.N. 4(c), 2070 'C']
   $\left(\text{Ans: } \frac{16}{33}\right)$
7. A card is drawn from a well–shuffled back of 52 cards. What is the probability that it is a king or a Diamond? 2 [Q.N. 4(c), 2070 'D']
   $\left(\text{Ans: } \frac{4}{13}\right)$

8. Two dice are rolled simultaneously. What is the probability of turning the same digit in both dice? 2 [Q.N. 4(c), Supp. 2069]

$\left(\text{Ans: } \frac{1}{6}\right)$

9. From 20 tickets marked from 1 to 20, one is drawn at random. Find the probability that it is a multiple of 4 or 5. [Q.N. 4(c), Set 'A' 2069]

$\left(\text{Ans: } \frac{2}{5}\right)$

10. A bag contains 5 red and 6 white balls. Two balls are drown at random. Find the probability that (i) both are red (ii) both are of the same colour. [Q.N.8(b), Set 'B' 2069]

$\left(\text{Ans: (i) } \frac{2}{11} \text{ (ii) } \frac{5}{11}\right)$

11. A card is drawn at random from a well-shuffled deck of 52 cards. What is the probability that is a red 8, a red 9 or a red 10? [Q.N. 4(c), 2068]

$\left(\text{Ans: } \frac{3}{26}\right)$

12. The chance that A can solve a certain problem is $\frac{3}{4}$. The chance that B can solve it is $\frac{2}{3}$, find the chance that the problem will be solved if they both try. [Q.N.8(b),2068]

$\left(\text{Ans: } \frac{3}{4}\right)$

13. Define mutually exclusive events and dependent cases with example while performing an experiment. [Q.N. 4(c), 2067]

14. A class consists of 40 boys and 60 girls. If two students are chosen at random, what will be the probability that (a) both are boys (b) both are girls (c) one boy and one girl ? [Q.N. 8(a), 2067]

$\left(\text{Ans: (a) } \frac{{}^{60}C_2}{{}^{100}C_2}, \text{(b) } \frac{{}^{40}C_2}{{}^{100}C_2}, \text{(c) } \frac{{}^{60}C_1 \times {}^{40}C_1}{{}^{100}C_2}\right)$

15. The chance that A can solve a problem is $\frac{1}{4}$, the chance that B can solve it is $\frac{2}{3}$. Find the probability that the problem will be solved if both of them try. [Q.N. 4(c), 2066]

$\left(\text{Ans: } \frac{3}{4}\right)$

16. Five men in a group of 20 are graduates. If three men are chosen out of 20 at random, what is the probability of at least one being graduates? [Q.N. 8(a)Or, 2066]

$\left(\text{Ans: } \frac{137}{22}\right)$

17. Given P(A) = 0.4, P(A ∪ B) = 0.56, P(B) = 0.3.
Are A & B independent ? [Q.N.4(c), 2065]
(Ans: not independent)

18. State and prove the theorem of compound probability [Q.N.8(a), 2065]

19. A bag contains 9 red, 7 white and 4 black balls. A ball is drawn at random. Find the probability of drawing (i) a white ball (ii) not a black ball. [Q.N. 4(c), 2064]

$\left(\text{Ans: (i) } \frac{7}{20}, \text{ (ii) } \frac{4}{5}\right)$

20. If P(A) and P(B) are the probabilities of the happening of the events A and B respectively, prove that : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
where $P(A \cup B)$ and $P(A \cap B)$ have the usual meanings. What will be the form of the above formula if A and B are independent events ? [Q.N. 8(a), 2064]

21. The chance that A can solve a certain problem is $\frac{1}{4}$ and the chance that B can solve it is $\frac{2}{3}$. Find the chance that the problem will be solved if they both try.

$\left(Ans.: \frac{3}{4}\right)$ [Q.N. 4(c), 2063]

22. If P(A) and P(B) be the probabilities of the independent events A and B respectively, prove that :

$$P(A \cap B) = P(A).\ P(B)$$

Where $P(A \cap B)$ has the usual meaning. [Q.N. 8(a), 2063]

23. What is the probability that an English alphabet selected at random is (i) a vowel (ii) a consonant ? [Q.N. 4(c), 2062]

$\left(Ans.: (i)\ \frac{2}{7}\ (ii)\ \frac{5}{7}\right)$

24. State and prove "The Theorem of Total Probability". [Q.N. 8(a), 2062]

25. A lot contains 10 items of which 3 are defective. Three items are chosen from the lot at random one after another without replacement. Find the probability that :

(i) All three are defective.

(ii) None of them are defective. [Q.N. 8(a)Or, 2062]

$\left(Ans.: (i)\ \frac{1}{120}\ (ii)\ \frac{119}{120}\right)$

26. State and prove the "Theorem of Compound Probability". [Q.N. 8(a), 2061]

27. A class consists of 60 boys and 40 girls. If two students are chosen at random, what will be the probability that (a) both are boys (b) both are girls (c) one boy and one girl ? [Q.N. 8(a) Or, 2061]

$$\left(Ans: (i)\ \frac{^{60}C_2}{^{100}C_2}\ (ii)\ \frac{^{40}C_2}{^{100}C_2}\ (iii)\ \frac{^{60}C_1 \infty\ ^{40}C_1}{^{100}C_2}\right)$$

28. Two dice are thrown. Determine the probability of getting a sum ≤ 5.

$\left(Ans: \frac{5}{18}\right)$ [Q.N. 4(c), 2060]

29. State and prove the "Theorem of total probability". [Q.N. 8(a), 2060]

30. Two letters are selected at random from the word "examination". Find the probability that both of them are same letters. [Q.N. 3(c), 2059]

$\left(Ans: \frac{3}{55}\right)$

31. If A, B, C are three mutually exclusive events with $\frac{1}{3}\ P(A) = \frac{2}{3}\ P(B) = \frac{1}{6}\ P(C)$, find P(A); P(B); and P(C). [Q.N. 8(a), 2059]

$\left(Ans: \frac{2}{7}, \frac{1}{7}, \frac{4}{7}\right)$

32. If A and B are two independent events with $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{5}$, find $P(A \cup B)$. $\left(Ans: \frac{13}{15}\right)$ [Q.N. 4(c), 2058]

33. State and prove the "Theorem of total probability". [Q.N. 8(a), 2058]