13. Mathematics I Paper (Math.311) Calculus

Exam 2066

Time: 3 hrs **Full Marks: 100**

Attempt ALL the questions.

Group 'A' **5×7=35**

1. Define and deduce the expressions for the polar subtangent and polar subnormal at any point $P(r,\theta)$ of a curve $r = f(\theta)$. Find the angle between the curves $r^2 = a^2\cos 2\theta$ and $r^2 = b^2\sin 2\theta$. [1+2+4]

2. State Taylor's series extended to infinity. Let R_n denote the remainder after n terms of the series. Prove that $\lim_{n\to\theta} R_n = 0$ is both necessary and sufficient condition that the function $f(x + h), |h| < \delta$ can be expanded in an infinite series. Hence show that $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$ to ∞ for all x. [1+3+3]

OR

State Leibnitz theorem.
If $y = \tan^{-1}x$, prove $(1 + x^2)y_1 = 1$ hence show that $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{y-1} = 0$

3. Define Beta and Gamma functions.
Prove that: $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$ and hence show that $\int_0^{\infty} e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$

4. How do you define the maximum and minimum values of a function of two variables? Find the minimum values of $x^2 + y^2 + z^2$ when $x + y + \ = 3a^2$ [2+5]

OR

State and establish Eulers theorem for a homogeneous function of degree n. Use this theorem to show that. $x\frac{2u}{2x} + y\frac{2u}{2y} = \tan u$ if $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ [1+2+4]

5. State the condition of exactness of a first order differential equation. Verify that the equation $(2xy + y^2)sx + (x^2 + 2xy - y)dy = 0$ is exact and hence find its general solution.

Group 'B'

10×4=40

6. What is the angle between the curve $r = \psi(\theta)$, $r = \phi(\theta)$? Show that the curves $ax^2 + by^2 = 1$ and $a^1x^2 + b^1y^2 = 1$ cut orthogonally if $\frac{1}{a} - \frac{1}{a^1} = \frac{1}{b} - \frac{1}{b^1}$ [1+3]

OR

Show that the tangent drawn at the extremities of any chord of the cardiode $r = (1 + \cos\theta)$ which passes through the pole are perpendicular to each other. [4]

7. What do you mean by inderterminate form? Evaluate $\lim_{x\to\frac{x}{2}}(\sin)^{\tan x}$ [1+3]

8. Evaluate $\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi} e^x \cos(y-x)dydx$ [4]

OR

So that $\int_0^1 dx\int_0^1 \frac{x-y}{(x+y)^3}dy \neq \int_0^1 dy\int_0^1 \frac{(x-y)}{(x+y)^3}dx$. [4]

9. Show tha t $\int_0^{\frac{\pi}{2}} \log \sin xdx = \int_0^{\frac{\pi}{2}} \log \cos xdx = \frac{\pi}{2}\log\left(\frac{1}{2}\right)$ [4]

10. Let the circle $x^2 + y^2 = a^2$ revolves round the x-axis, show that the volume of the whole sphere generated is $\frac{4}{3}\pi a^3$ [4]

11. Obtain the reduction formula for $\int \cos ec^n xdx$ and find $\int cox\ ec^5\ xdx$. [4]

OR

Find from definition, the value of $\int_a^b e^x dx$. [4]

12. Find the complete solution and the singular solution of the equation $y = px + p - p^2$. [4]

13. Find the complimentary function and particular integral of the differential equation $\frac{d^2y}{dx^2} + a^2y = \text{sec}ax$. [1+3]

OR

Solve: $(D^2 + 3X + 2)y = e^{2x}$, given that $y = 0, \frac{dy}{dx} = 0$ where $x = 0$ [4]

14. Solve: $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ [4]

15. Define Clairaut equation and solve $y = px + \sqrt{a^2p^2+b^2}$. [4]

Exam 2067

Time: 3 hrs

Full Marks: 100

Attempt ALL the questions.

Group 'A'

5×7=35

1. Define the length of perpendicular from the pole on the tangent to a curve. Also define pedal equation and obtain its expression for the curve $r^2 = a^2 \cos 2\theta$. [3+1+2]

OR

What is the pedal equation of a curve? Deduce its equation from Cartesian equation. Find the Geometrically the pedal equation of the ellipse with respect to focus. [1+3+3]

2. State Rolle's theorem and give its geometrical meaning. Verify that the function y = f(x) = x^2 4x + 3 satisfies the Rolle's theorem in the interval $1 \le x \le 3$ and hence find the number c such that f' (c) = 1. [1+2+3+1]

3. What do you mean by term 'integration'? Explain it. If f(x) is continuous in the interval (a, b), b > a, show that the integral $\int_a^b f(x)dx$ geometrically represents the area of the space enclosed by the curve y = f(x), the ordinates x = a, x = b and the x –axis. [2+5]

OR

State fundamental theorem of integral calculus.

Evaluate: $\int_a^b \frac{1}{x}dx$ as the limit of a sum. [1+6]

4. State and establish Euler's theorem for homogeneous function of degree n. Use this theorem to show that

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$ if $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ [1+2+4]

5. Define a differential equation of the second order. What do you mean by complimentary function and the particular integral? Solve : $\frac{d^2y}{dx^2} + a^2y = \sec ax$.[2+5]

Group 'B' **10×4=40**

6. Prove that the sum of the intercept of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant. [4]

OR

Define the Cartesian subtangent and subnormal at any point of a curve. Show that for the curve $by^2 = (x + a)^3$, the square of the subtangent varies as the subnormal. [1+3]

7. State L'Hospital's riule.

Show that : $\lim_{x\to 0}(\cos x)^{\cot x} = e^{-\frac{1}{2}}$ [1+3]

8. Evaluate: $\int_1^2 dy \int_3^4 \frac{dx}{(x+y)^2}$ [4]

OR

Show that $\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3}dx \neq \int_0^1 dy \int_0^1 \frac{s-y}{(x+y)^3}dx$ [4]

OR

9. Define Beta function. Show that for m > –1,n>–1.

$\int_a^b (x-a)^m(b-x)^n dx = (b-a)^{m+n+1}\beta(m+1,n+1)$. [1+3]

OR

Prove that $\int_0^\infty \sqrt{y}e^{-y^2} dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}dy = \frac{\Pi}{2\sqrt{2}}$ [4]

OR

10. Find the area of a loop of a curve r = asin3θ. [4]

11. Show that $\int_0^{\pi} \frac{x\tan x}{\sec x + \cos x}dx = \frac{\pi^2}{4}$ [4]

12. If $y = (\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$ and hence show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+2} - x^2y_n = 0$. [4]

13. What do you mean by linear differential equation of first order? Solve: $\cos x\frac{dy}{dx} + y\sin x = \sec^2 x$. [1+3]

14. State Clairaut's equation.
Find the general solution of $y = 2px + y^2p^3$.

OR

Find the general and singular solution of $y = px + \sqrt{a^2p^2 + b^2}$ [4]

15. Find the particular integral $\frac{1}{f(D^2)}\sin ax$. Where $f(D^2) = D^2 + a^2$. [4]

Exam 2068

Time: 3 hrs **Full Marks: 100**

Attempt ALL the questions.

Group 'A' 5×7=35

1. What the pedal equation of a curve is? Deduce its equation from Cartesian equations.
Find geometrically the pedal equation of the ellipse with respect to the focus. [1+3+3]

2. State the criteria for a function of two variables to have maximum and minimum values. Find the maximum and minimum values of the function $x^3 + y^3 - 3axy$. [2+5]

OR

Sate and establish Euler's theorem for a function of two variables. Use this theorem to prove that
$x\frac{2u}{2x} + y\frac{2u}{2y} + \frac{1}{2}\cot u = 0$, where $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ [1+3+3]

3. What do you mean by indeterminate form? State various forms of indeterminacy.
Evaluate: $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. [1+2+4]

OR

What do you mean by the curvature and radius of curvature of a curve? Show that the circle is a curve of uniform curvature, and its radius of curvature at every point is constant. [2+1+4]

4. State and prove Lagrange's Mean value theorem and give its geometrical meaning.
Find the value of θ in the mean value theorem.
$f(x + h) = f(x) + hf'(x + \theta h)$ if $f(x) = 1/x$. [1+3+3]

5. Show that the necessary and suffcient condition for the differential equation of the form $Mdx + Ndy = 0$, where M and N are function of x and y to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Verify that the equation $(2x3+ + 4y)dx + (4x + y - 1)\,dy$ is exact and obtain its general solution. [3+1+3]

Group 'B' [10×4=40]

6. Define an aysmpotte to a curve. Find asymptote to a surve:
$x(x-y)^2 - 3(x^2 - y^2) + 8y = 0$. [1+3]

7. If $y = e^{a\sin^{-1}}$ then prove tht
a) $(1-x^2)y_2 = xy_1 + a^2y$

b) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}(n^2 + a^2)y_n = 0.$ [1+3]

OR

Sate Rolle's theorem. Verify the theorem for the function $f(x) = (x-2)(x-3)(x-4)$ in [2,4]. [1+3]

8. Show that: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{xinx}dx}{\sqrt{\sin x} + \sqrt{\cos x}} = \frac{\pi}{4}.$ [4]

9. If f(x,y) is homogeneous function of x & y of degree n, prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x,y).$ [4]

OR

State Euler's theorem on homogeneous function of two independent variables. Verify it for $u = x^n \tan^{-1}(y/x)$. [1+3]

10. Define a Gamma function.

Prove that: $\Gamma(m)\,\Gamma(1-m) = \frac{\pi}{\sin m\pi}(0<m<1).$ [1+3]

11. Find the surface area of the solid generated b y revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1+\cos\theta)$ about its base. [4]

OR

Find the area included between the two parabolas $y^2 = 4ax$ and $x^2 = 4ay$. [4]

12. Prove by evaluating the repeated integrals, that [4]

$$\int_0^1 dx \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2} dy \neq \int_0^1 dy \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2} dx.$$ [4]

13. Find the solution of the equation the: $(y + 1)p - xp^2 + 2 = 0.$ [4]

OR

State Clairuat's equation and show that how it provides complete solution and singular solution.

14. Solve $(D^2 + 1)y = \sin 2x_1$ when $x = 0$, $y = 0$ and $y_1 = 0$. [4]

15. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}.$ [4]

Exam 2069

Time: 3 hrs **Full Marks: 100**

Attempt ALL the questions.

Group 'A' **5×7=35**

1. State MaClaurin's series in finite form. What is the condition under which this series can be extended to infinity? Hence or otherwise show that

$\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \ldots\ldots\ldots$ [1+2+4]

2. What do you mean by the curvature of a curve? Hence define the radius of curvature.

Show that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ the radius of curvature at any extremity of the major axis is equal to half of latus rectum. [1+1+5]

OR

Describe the curve tracing techniques for a given Cartesian equation. Trace the curve of the function. $x^2y^2 - x^2 + a^2 = 0.$ [4+3]

3. Define Beta and Gamma function.

Prove that $\int_0^{\frac{\pi}{2}} \sin^p x \cos^q(x)dx = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$, p,q>–1. And hence show that

$\int_0^{\infty} e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$ [2+3+2]

4. What do you mean by Lagrange's undetermined multipliers? Use Lagrange's multiplier to find the minimum value of $x^2 + t^2 + z^2$ subject to the constraint x + y + z = 3a. [2+5]

OR

State and establish Euler's theorem for a function of two variables. Use this theorem to show that

$x\frac{\partial y}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20}u$, where $u = \frac{x^{\frac{1}{4}}+u^{\frac{1}{4}}}{x^{\frac{1}{5}}+u^{\frac{1}{5}}}$ [1+3+3]

5. State Clairaut's equation and show that how Clairaut's equation provided complete solution and singular solution. Find the completer and singular solutions of the differential equation $y = px + p - p^2$. [1+3+3]

Group 'B' 10×4=40

6. What do you mean by pedal equation of a curve? Find the pedal equation of the curve r = a(1 + cosθ). [1+3]

OR

Define the polar subtangent and polar subnormal at any point of a curve r = f(θ). Show that for the curve $r = e^{\theta}$, the polar subtangent is equal to the polar subnromal. [2+2]

7. If y = a cos (log x) + b sin(log x), then show that [4]
$x^2y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$/ [4]

8. State L'Hospital's rule. Evaluate $\lim_{x\to\frac{\pi}{4}}(\tan x)^{\tan 2x}$. [1+3]

9. Prove that $\int_0^{\alpha} f(x)dx = \int_0^{\alpha} f(a-x)dx$ and use it, prove taht $\int_0^{\pi}\frac{x\sin x\, dx}{1+\cos^2 x} = \frac{\pi^2}{4}$ [1+3]

10. Obtain a reduction formula for $\int \sin^n x dx$ and find $\int \sin^4 x\, dx$.. [4]

Define Gamma function.

Use it, prove that $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}} = \frac{\pi}{3}$ [1+3]

11. Find the perimeter of the asteroid formed by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis if $\frac{4\pi ab^2}{3}$. [4]

12. Evaluate: $\int_0^{\frac{\pi}{2}} dy \int_0^{\frac{\pi}{2}} \cos(x + y)\, dx$. [4]

13. Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [4]

14. Solve: $(D^2 - 3D + 2)y = e^x$ where $D = \frac{d}{dx}$. [4]

OR

Solve: $(D^2 + 4)y = \sin 2x$. [4]

15. Solve: $(D^2, + 1) = \cos 2x$, when $x = 0$, $y = 0$ and $y_1 = 0$. [4]

Exam 2070

Full Marks: 75

Time: 3 hrs.

New Course

Attempt all the questions.

Group "A"

[7×5=35]

1. Find angle of intersection of two curves whose polar equations are $r=f(\theta)$ and $r=d(\theta)$. What happens if the two curves touch? Find the angle between the curves $r=2 \sin \theta$ and $r=2 \cos \theta$

2. State Cauchy's mean value theorem and state when it reduces to Lagrange's mean value theorem, verify Lagrange's mean value theorem, verify Lagrange's mean value theorem for the function $f(x)=x(x-1)(x-2)$ in the interval [0.½]. Also show that the function f(x) is concave downward at $x=\frac{1}{5}$ [1+½+4+1½]

3. What do you mean by the asymptotes to a curve? Obtain the condition that a line $y= mx +c(m \neq 0)$ is a asymptote to the curve $f(x, Y)=0$. Find the asymptote of $x^3-y^3=3ax^2$ [1+3+3]

OR

What do you mean by the curvature of a curve at a point? Show that the circle is a curve of uniform curvature. Find the radius of curvature of the curve $r^m= a^m \cos m\theta$ [1+3+3]

4. Show that the area of the region bounded by a curve $y=f(x)$, the axis of x, and two ordinates $x=a$ and $x= b$ is $\int_a^b f(x)\,dx$ and hence find the area bounded by the parabola $y^2=4ax$ and its latus rectum. [3+4]

5. Show that the necessary and sufficient condition for the differential equation of the form $Mdx + Ndy = 0$, where M and N are functions of x and y to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Verify that the equation $(x^2+xy^2)dx+(x^2y+y^2)dy=0$ is exact and obtain its general solution. [3+1+3]

OR

State the homogenous equation of the first order and first degree. Is the equation $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$ homogeneous? If not make it homogeneous and solve it. [1/2+1/2+2+4]

Group "B"

[10×4=40]

6. If $y = a \cos(\log x)+b \sin(\log x)$, then show that $x^2y_{n+2}+(sn+1)xy_{n+1} +(n^2+ 1)y_n=0$. [4]

7. Finds the extreme value of the function x^2+y^2 under the condition $x+4y=2$. [4]

OR

If $u = \log \frac{x^4+y^4}{x+y}$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{34}{e^4}$ [4]

8. Show that $\int_0^{\pi/2} \log \sin x\, dx = \int_0^{\pi/2} \log \cos x\, dx = \frac{\pi}{2} \log \frac{1}{2}$. [4]

9. Prove that $\int_0^{\infty} e^{-ax} \cos bx\, dx = \frac{a}{a^2+b^2}$ a>0. [4]

OR

Obtain a reduction formula for $\int \sec^n x\, dx$ and hence find $\int \sec^6 x\, dx$.

10. Find the area bounded by the cycloid $x=a(\theta+\sin\theta)$ $y=a(1-\cos\theta)$ and its base.

OR

Find the perimeter of the asteroid $x^{2/3}+y^{2/3}=a^{2/3}$

11. Evaluate $\int_0^{4a} dx \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dx$ [4]

12. Solve $(D^2+2D+1)y=c^x+c^{-x}$, where $D=\frac{d}{dx}$ [4]

13. Find the complete primitive and singular solution of $y=px+ap(1-p)$ where $P=\frac{dy}{dx}$ [4]

14. Find the equation of the curve for which the sum of the reciprocals of the radius vector and the polar subtangent is constant. [4]

OR

What do you mean by an initial condition for a differential equation?

Solve : $y(1-x^2)\frac{dy}{dx} + x(1-y^2)=0$, give that y=1 when x=0 [1+3]

15. State L' Hospital's rule and use lit to evaluate

$\lim_{x\to 0} (\cos x)^{\cos 2x}$ [1+3]

Old course

Attempt all the questions.

Group "A" [5×7=35]

1. What is the pedal equation of a curve? Deduce is equation from Cartesian equation.

Find geometrically the pedal equation of the ellipse with respect to focus. [1+3+5]

2. State Rolle's Theorem and interpret lit geometrically.

If f (x)=0 for all values of x in an interval, then show that f(x)) is constant in the interval. [1+2+4]

OR

State Cauchy mean value theorem and state when it reduces to Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function.

f(x)=x (x–1) (x–2) in the interval [0.½]

Also, who that the function f(x) is concave downward at $x=\frac{1}{5}$ [1+½+4+1½]

3. What do you mean by the indeterminate form? State various forms of indeterminancy. When do you apply L'Hospitals rule? Evaluate lint lim

$\lim_{x\to 0} (\cos x)^{1/x^2}$

4. Define Beta and Gamma functions. Prove that $1(\frac{1}{2}) = \sqrt{\pi}$ and use Gamma function to prove that $\int_0^1 \frac{dx}{(a-x^6)^{1/6}} = \frac{\pi}{3}$ [1+3+3]

5. Define a differential equation of the second order. What do you mean by complementary function and the particular integral?

Solve : $\frac{d^2y}{dx^2} + a2y = \sec ax$ [1+2+4]

OR

Define auxiliary equation of the differential equation of the second order. If the auxiliary equation has two equal roots say a, may its general solution be $y = c\, e^{ax}$? If not why? And deduce the correct general solution.

Solve : $(D^2+2D+5)\, y = 0$, where $D = \frac{d}{dx}$ [1+1+3+2]

Group "B"

[10×4=40]

6. What are polar subtangent and polar subnormal? Show that for the curve $r\theta = a$, the polar subtangent is constant and for the curve $r = a\theta$, the polar subnormal is constant. [1+3]

7. Define centre of a curvature of a curve. Fine the centre of curvature at any point (x, y) on the parabola $y^2 = 4ax$. [1+3]

OR

Show that the chord of curvature parallel to y – axis for the curve $y = c \cosh \frac{x}{c}$ is double of the ordinate. [4]

8. Trace the curve $y = x^3 - 12x - 6$ [4]

9. Show that $\int_0^{\pi} \frac{x \sin x\, dx}{1+\cos^2 x} = \frac{\pi^2}{4}$ [4]

10. Find the perimeter of the cardioid $r = a(a+\cos\theta)$ and show that the arc of the upper half is bisected by $\theta = \frac{\pi}{3}$

OR

Find the area of a loop of the curve $a^2y^2 = a^2x^2 - x4$

11. Examine where the equation $x\,dx + y\,dy + (x^2+y^2)\,dy = 0$ is exact or not and hence solve it. [1+3]

12. State Ruler's theorem on homogeneous function of two independent variables. Verify it for $u = (x^2+y^2)^{1/3}$ [1+3]

OR

Find $\frac{dy}{dx}$ of $(\tan x)^y + (y)^{\tan x} = 0$. [4]

13. Find the general and singular solution of $y = px + p(1-p)$. [4]

14. Define linear differential equation of the first order.

Solve : $\frac{dy}{dx} + y \tan x = \sec x$. [1+3]

15. Solve: $(D^2+D-2)\, y = e^{2x}$, given that when x=0, y=0 and $y_1=0$. [4]

OR

Solve: $(D^2+16)y = \cos 4x$.

14. Mathematics II (Math.312) Analytical Geometry & Vector Analysis

Exam 2066

Time: 3 hrs

Full Marks: 75

Attempt ALL the questions.

Group 'A' **5×7=35**

1. When does the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a parabola, an ellipse and hyperbola? What conic does the equation $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ represent? Find the center of the conic. [3+4]

OR

Define that tangent to a conic.
$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) obtain the condition that the line $lx + my + n = 0$ may be a tangent to the conic $s = 0$ at (x_1, y_1). [2+5]

2. Define the auxiliary circle and eccentric angle of a point with respect of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [1+1+5]
Find the foci, directrices, accentricity, the ends of latus rectum and length of latus rectum of the ellipse $9x^2 + 25y^2 = 225$.

3. Define skew lines and line of shortest distance. Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$. Find also the equation of shortest distance. [1+1+5]

4. Define a great and small circle of sphere. Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. [1+1+5]

OR

Define tangent line and tangent plane at a point of the sp[here. Find the equation to the spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$. [1+1+5]

5. Define scalar triple product and prove geometrically that the scalar triple product represents the volume of the parallelopeped. Also verify that in the scalar triple product position of dot and cross can be interchanged. [1+3+3]

Group 'B' **10×4=40**

6. What is the equation $(x - a)^2 + (y - b)^2 = c^2$ become when it is transformed to parallel axes through the point $(a - c, b)$? [4]

OR

Find the polar coordinates of the points (3,4,5) and (–2,1,2). [2+2]

7. Show that the line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$. [4]

OR

Show that the tangent at the extremity of any diameter of an ellipse is parallel to the chords which it bisect.

8. State the condition under which the general equation of second degree $ax^2 = 2xhy + by^2 + 2gx + 2fy + c = 0$ may represent an ellipse. Find the center of the conic section $2x^2 - 5xy - - 5xy - 3y^2 - x - 4y + 6 = 0$ and its equation when transformed to the center. [1+3]

9. Find the equation of the plane through the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ parallel to the line $\frac{x}{l'} = \frac{y}{m'} = \frac{z}{n'}$. [4]

10. Show that the equation to a right circular cone whose vertex is 0, axes OX and semivertical angle α' is $y^2 + z^2 = x^2\tan^2\alpha$. [4]

OR