14. Mathematics II (Math.312) Analytical Geometry & Vector Analysis

Exam 2066

Time: 3 hrs **Full Marks: 75**

Attempt ALL the questions.

Group 'A' **5×7=35**

1. When does the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a parabola, an ellipse and hyperbola? What conic does the equation $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ represent? Find the center of the conic. [3+4]

OR

Define that tangent to a conic.
$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) obtain the condition that the line $lx + my + n = 0$ may be a tangent to the conic $s = 0$ at (x_1,y_1). [2+5]

2. Define the auxiliary circle and eccentric angle of a point with respect of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. [1+1+5]
Find the foci, directrices, accentricity, the ends of latus rectum and length of latus rectum of the ellipse $9x^2 + 25y^2 = 225$.

3. Define skew lines and line of shortest distance. Find the shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$. Find also the equation of shortest distance. [1+1+5]

4. Define a great and small circle of sphere. Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. [1+1+5]

OR

Define tangent line and tangent plane at a point of the sp[here. Find the equation to the spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$. [1+1+5]

5. Define scalar triple product and prove geometrically that the scalar triple product represents the volume of the parallelopeped. Also verify that in the scalar triple product position of dot and cross can be interchanged. [1+3+3]

Group 'B' **10×4=40**

6. What is the equation $(x - a)^2 + (y - b)^2 = c^2$ become when it is transformed to parallel axes through the point $(a - c, b)$? [4]

OR

Find the polar coordinates of the points (3,4,5) and (–2,1,2). [2+2]

7. Show that the line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$. [4]

OR

Show that the tangent at the extremity of any diameter of an ellipse is parallel to the chords which it bisect.

8. State the condition under which the general equation of second degree $ax^2 = 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent an ellipse. Find the center of the conic section $2x^2 - 5xy - - 5xy - 3y^2 - x - 4y + 6 = 0$ and its equation when transformed to the center. [1+3]

9. Find the equation of the plane through the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ parallel to the line $\frac{x}{l'}=\frac{y}{m'}=\frac{z}{n'}$. [4]

10. Show that the equation to a right circular cone whose vertex is 0, axes OX and semivertical angle α' is $y^2 + z^2 = x^2\tan^2\alpha$. [4]

OR

Define reciprocal cone.

Prove that the cone $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal.

11. Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generators parallel to the line $x = y = z$. [4]
12. Obtain the condition that the plane $lx + my + nz = p$ may touch the central conicoid $ax^2 + by^2 + cz^2 = 1$. [4]
13. Prove the folloiwng;

a) $\left(\vec a \times \vec b\right)^2 = a^2b^2 - \left(\vec a . \vec b\right)^2$.

b) If $\vec a + \vec b + \vec c = 0$, then $\vec a \times \vec b \times \vec c = \vec c \times \vec a$ [4]

14. If $\vec r = a\cos t\ \vec i + a\sin t\ \vec j + at\tan\alpha\ \vec k$ find $\left|\frac{d\vec r}{dt} \times \frac{d^2\vec r}{dt^2}\right|$ and $\left[\vec r\ \frac{d\vec r}{dt} \times \frac{d^2\vec r}{dt^2}\right]$
15. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2).

OR

Prove that curl (grad ϕ) = 0. [4]

Exam 2067

Time: 3 hrs **Full Marks: 75**

Attempt ALL the questions.

1. What is conic section? When it becomes hyperbola? Obtain the length of the axes,, eccentricity, coordinates of foci, equation of directrix and length of latus rectum of the hyperbola. $16x^2 - 25y^2 = 400$. [1+1+5]
2. What are the conditions under which the second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent (i) a hyperbola (ii) an ellipse (iii) a parabola? What conic does the equation $12x^2 - 23xy + 10y^2 - 25x + 26y - 14 = 0$ represent? If possible, find the center and its equation referred to the circle. [1+1+3+2]

OR

Define pole and polar with respect to conic. Obtain the equation of polar of any point (x', y') w. r. t. to the conic represented by $ax^2 + 2xhy + by^2 + 2gx + 2fy + c = 0$. [2+5]

3. Find the equation of straight line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2). Find the point where the line joining (2, 1, 3) and (4, –2, 5) cuts plane $2x + y - z - 3 = 0$. [3+4]
4. Define reciprocal cone. Prove that the equation $\sqrt{fx} + \sqrt{gy} + \sqrt{hz} = 0$ represent a cone which touches the coordinate planes and that equation of the reciprocal cone is $fyz + gzx + hyx = 0$. [1+6]

OR

Define a cone. Obtain the equation of cone with vertex (α, β, γ) and base the parabola $z^2 = 4ax$, $y = 0$ [1+6]

5. Define vector triple product of any non zero vectors $\vec a, \vec b, \vec c$ and give its geometrical meaning. Find an expression for $\vec a \times \left(\vec b \times \vec c\right)$. [1+2+4]

Group 'B' **10×4=40**

6. What does the equation $2x^2 + y^2 - 4x + 4y = 0$ become, when it is transferred to parallel axis through the point (1, –2). [4]

OR

Find the distances of the pint (1,2,3) from the coordinate axes. Also find its distance from the origin. [3+1]

7. If e and e' the eccentricity of hyperbola and its conjugate prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$
8. What is a focal chord or a conic? In any conic prove that the sum of the reciprocal of the segments of any focal chord s constant. [1+3]

9. Find the equation of plane through (2, –3, 1) normal to the line joining (3, 4, –1) and (2, –1, 5). [4]

10. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinates axes at A, B, C. Prove that the equation of the cone generated by the lines drawn from O to meet the circle ABC is $yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0.$ [4]

OR

Show that the equation to a right cone whose vertex is O, axis OX and semi-vertical 'α' is $y^2 = x^2 \tan^2\alpha$. [4]

11. Find the equation of the sphere which passes through the origin and the points (0, 1, –1), (–1, 2, 0) and (1, 2, 3). [4]

OR

Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.

12. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point (α, β, γ). Prove that the perpendiculars to them form the origin generates the cone $(\alpha x, \beta y, \gamma z)^2 = a^2x^2 + b^2y^2 + c^2z^2$. [4]

13. Show that: $\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ [4]

14. Prove that: $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

15. The necessary and sufficient condition for the vector function of a scalar variable to have a constant magnitude is $\vec{a}.\frac{d\vec{a}}{dt}=0$. [4]

OR

If $\vec{a}$ is a constant vector then prove that. (i) $\vec{a}.\nabla\left(\frac{1}{r}\right)=-\frac{\vec{a}.\vec{r}}{r^3}$ (ii) grad $\left(\vec{a}.\vec{a}\right)=\vec{a}$ [2+2]

Exam 2068

Time: 3 hrs **Full Marks: 75**

Attempt ALL the questions.

Group 'A' **5×7=35**

1. What type of the conic section is the hyperbola? Define its foci and eccentricity and directrix.
Determine the center, coordinates of foci, the eccentricity, length of the latus rectum and the equation of the directrices of the hyperbola. $5x^2 - 6y^2 = 30$. [1+2+4]

2. What conic does the equation $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$ represent? If possible, find the center and its equation referred to the center. [2+5]

OR

Define pole and polar with respect to conic. Determine the equation of the polar with respect to the conic represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [2+5]

3. Define the skew lines and the line of shortest distance. Find the shortest distance between the lines. $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4}$ and $\frac{x-4}{3}=\frac{y-5}{4}=\frac{z-7}{5}$. Also find the equation of the shortest distance. [1+4+2]

4. What do you mean by a great circle and a small circle of the sphere? Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle, determine its center and radius. [1+1+5]

OR

Find the tangent line and tangent plane at a point of a sphere. Show that the plane $2x - y + 3z = 14$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$. Find the point of contact. [2+3+2]

5. Define reciprocal system of vectors. If $\vec{a'}$, $\vec{b'}$, & $\vec{c'}$ be reciprocal system to three on coplanar vectors $\vec{a}$, $\vec{b}$,& $\vec{c}$, then prove the followings:

i) $\vec{a} . \vec{a'} = \vec{b} . \vec{b'} = \vec{c}\ \vec{c'} = 1$

ii) $\vec{a} . \vec{b'} = \vec{b} . \vec{c'} = \vec{c} . \vec{a'} = 0$

iii) $[\vec{a'}\ \vec{b'}\ \vec{c'}][\vec{a}\ \vec{b}\ \vec{c}] = 1$ [1+1+1+4]

Group 'B' 10×4=40

6. If the axes be turned through an angle $\tan^{-1}(2)$ what does the euqation $4xy - 3x^2 = a^2$ becomes? [4]

7. Find the locus of the point of intersection of the tangents to the ellipse which meet at right angles. What is the nature of the locus? [3+1]

OR

Show the line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$. [4]

8. Find the equation of the polar with respect to the conic represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [4]

OR

Prove that the equation $\frac{l}{r} = 1 - e\cos\theta$ and $\frac{l}{r}\ e\cos\theta - 1$ represent the same conic.

9. Find the equation of the plane through (–1, 1, –1) and (6, 2, 1) normal to the plane $2x + y + z = 5$. [4]

10. Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ [4]

11. Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generators parallel to the line $x = y = z$. [4]

12. Obtain he condition that the plan $lx + my + nz = p$ may touch the central conicoid $ax^2 + by^2 + cz^2 = 1$. [4]

OR

Find the equations of the planes which contain the line given by $5x + 6y - 18 = 0$ and $3y - z = 0$ and touch the ellipsoid $5x^2 + 3y^2 + z^2 = 36$.

13. If $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} + \vec{j}$ and $\vec{c} = \vec{i} + 2\vec{j} - \vec{k}$ verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$.

14. show that the necessary and sufficient condition for the vector function $\vec{a}$ of a scalar variable t to have constant magnitude is $\vec{a}.\frac{d\vec{a}}{dt} = 0$ [4]

OR

A particle moves along the curve $x = 4\cos t$ $y = 4\sin t$, $z = 6t$. Find the magnitude of acceleration at time $t = \pi$.

15. Define curl of a vector function. If ϕ be a scalar function prove that curl (grad ϕ)=0. [1+3]

Exam 2069

Time: 3 hrs **Full Marks: 75**

Attempt ALL the questions.

Group 'A' 5×7=35

1. Define conic section. Find the center, foci, eccentricity, latus rectum and length of axes of the ellipse $x^2 + 4y^2 - 4x + 24y + 24 = 0$. [1+6]

2. Define general equation of second degree and show that general equation of second degree in x & y represent a conic section. [1+6]

OR

Find the center of the conic $9x^2 - 4xy + 6y^2 - 14x - 8y + 1 = 0$ show that this conic is a ellipse. Also find its semi-axes and eccentricity. [7]

3. Define shortest distance between the lines. Obtain the equation of the line of shortest distance between the lines. [1+6]

4. Define a cone. Determine the equation of the cone with vertex (α, β, γ) and base $y^2 = 4ax, z = 0$. [1+6]

OR

Define the generator of a cone. Find the condition that the cone has three mutually perpendicular generator. [1+6]

5. Define reciprocal system of vectors. If $\vec{a}', \vec{b}', \& \vec{c}'$ be reciprocal system to three on coplanar vectors $\vec{a}, \vec{b}, \& \vec{c}$, then prove the followings:

i) $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 1$

ii) $\vec{a}.\vec{b}' = \vec{b}.\vec{c}' = \vec{c}.\vec{a}' = 0$

iii) $[\vec{a}'\ \vec{b}'\ \vec{c}'][\vec{a}\ \vec{b}\ \vec{c}] = 1$ [1+1.5+1.5+3]

Group 'B' 10×4=40

6. What does the equation $(x - a)^2 + (y - b)^2 = c^2$ become when it is transferred to parallel axes through the point (a, b, -c)? [4]

7. Define normal to the ellipse. Prove that the straight line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$. [4]

OR

Show that the tangent at the extremity of any diameter of an ellipse is parallel to the chords which is bisect. [4]

8. Find the center of the conic section. $9x^2 - 4xy + 6y^2 - 14x + 8y + 1 = 0$ [4]

9. Find the point where the line joining (2, 1, 3)(4, –2, 5) cuts the plane $2x+y-z-3= 0$. [4]

OR

Find the equation of line through the point (2, 3, 1) and parallel to the planes $2x + 3y + 4z = 5$ and $32x + 4y + 5z = 6$. [4]

10. Find the shortest distance between the lines $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}$ [4]

11. A variable plane is parallel to the given plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in A, B, C. Prove that the circle ABC lies on the cone

$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0.$ [4]

12. Planes through OX and OY include an angle α. Show their line of intersection lines on the cone. $z^2(x^2 + y^2 + z^2) = x^2y^2 \tan^2\alpha$. [4]

OR

Prove that the cone $ayz + bzx + cxy = 0$ and $\sqrt{ax} + \sqrt{by} + \sqrt{cz} = 0$ are reciprocal.

13. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are the reciprocal system of vectors prove that $\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' = 3$. [4]

14. If $\vec{r_1} = 2t^2\vec{i} + 3(t-1)\vec{j} + 4t^2\vec{k}$ and $\vec{r_2} = (t-1)\vec{i} + t^2\vec{j}\,(t-2)\vec{k}$, show that $\int_0^2 (\vec{a}, \vec{a})\, dt = \frac{4}{3}$ [4]

15. Prove curl (grad ϕ) = 0 [4]

OR

If $\vec{f} = x^2z\vec{i} - 2y^3z^2\vec{j} + xy^2z\vec{k}$

find div $\vec{a}$ and curl $\vec{a}$ at (1, –1, 1). [4]

Exam 2070

New Course

Full Marks: 75
Time: 3 hrs.

Attempt all the questions.

Group "A" [5×7=35]

1. What type conic section is the hyperbola? Find the coordinates of centre, foci, equation of directrix, eccentricity and lotus rectum of the hyperbola.
$4x^2-9y^2+8x+18y-41=0$ [1+6]

OR

Define conjugate hyperbola, Give an example to show that a hyperbola and its conjugate have the same asymptotes. Find the equation to the hyperbola, whose asymptotes arc the straight lines $x+2y+3=0$ and $3x+4y+5=0$ and which passes through the point $(1,-1)$ [1+1+5]

2. Define tangent and normal to a curve. Find the condition that any straight line $x+my+n=0$ may touch the conic
$ax^2+2hxy+by^2+2gx+2fy+c=0$.[2+5]

3. What are skew lines and line of shortest distance?
Find the shortest distance between the line $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{5}=\frac{z+2}{2}$
Also find the equation of the shortest distance. [1+4+2]

4. Find the equation of the tangent plane at (α,β,γ) to the conicoid $ax^2+by^2+cz^2=1$ and hence write down the equation of the tangent plane at a point (α,β,γ) of the ellipsoid. [5+2]

OR

What is conicoid? Give the condition under which it represents an ellipsoid. Show that six normal's can be drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}=1$ from any given point. [1+1+5]

5. Define derivative of a vector function of a scalar variable. Prove that the necessary and sufficient condition for the vector function a of a scalar variable to have a constant magnitude is $a\frac{da}{dt}=0$
A particle moves along the curve $x=2\sin 35$, $y=2\cos 3t$, $z=8t$, find the magnitude of the velocity at $t=\frac{\pi}{3}$ [1+4+2]

Group "B" [10×4=40]

6. Find the equation of the curve $9x^2+4y^2+18x-16y=11$ referred to parallel axes through $(-1,2)$, [4]

7. Find the equation to the common tangents of the circle $x^2+y^2=4ax$ and the parabola $y^2=4ax$. [4]

OR

Show that the line $ex+my=n$ is normal to the ellipse.
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $\frac{a^2}{e^2}+\frac{b^2}{n^2}=\frac{(a^2-b^2)^2}{n^2}$

8. Prove that the equation $\frac{l}{r}=1+e\cos\theta$ and $\frac{l}{r}=-1+e\cos\theta$ represents the same conic.

9. Find he equation of the plane through $(2,-3,1)$ normal to the line joining $(3,4,-1)$ and $(2,-1,5)$

OR

Show that the equation of plane through (–1, 1, –1) and (6, 2, 1) normal to the plane $2x + y + z = 5$

10. Show that the equation to a right circular cone whose vertex is 0, axis OX and semivertical angle 'α' is $y^2 + z^2 = x^2 \tan^2 \alpha$.

11. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 1$, $2x + 4y + 5z = 6$ and touching the plane $z = 0$

12. If 2r be the distance between two parallel tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, prove that the line through the line through the origin pertpendicular to the plane lies on the cone $(a^2-r^2)x^2 + (b^2-r^2)y^2 + (c^2-r^2)z^2 = 0$

OR

Find the equation of the tangent plane at (α, β, γ) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

13. Show that : $[\vec{l} + \vec{m}\ \vec{m} + \vec{n}\ \vec{n} + l] = [\vec{l}\ \vec{m}\ \vec{n}]$

14. If $\vec{r} = a\cos t\ \vec{i} + a\sin t\ \vec{j} + a\tan a\ t\ k$, find $\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right|$ and $\left[\vec{r}\ \frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\right]$

15. Define divergence and curl of a vector function if $\phi = \log(x^2+y^2+z^2)$, find curl (grad ϕ)

OR

Defince curl of a vector function if ϕ be a sealar function prove that curl (grad ϕ)=0

Old Course

Attempt all the questions:

Group "A" **[5×7=35]**

1. Define conic section. When does it become ellipse? Obtain the length of axes, the eccentricity, the coordinates of foci, the length of latus rectum, and the equation of directories' of the ellipse $9x^2+25y^2=225$. **[1+1+5]**

2. State the conditions under which the general equation of second degree may represent (i) a parabola (ii) an ellipse (iii) a hyperbola. What conic does the equation $2x^2-72xy+23y^2-4x-28y-48=0$ represent? If possible, fine the centre and its equation referred to the centre. **[1+1+3+2]**

OR

Define normal to a curve. Obtain the equation of the normal at any point (x' y') of the conic represented by $ax^2+2hxy+by^2+2gx+sfy+c=0$. **[1+6]**

3. Find the equation of straight line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2), Find K so that the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ = and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ may be perpendicular to each other. **[3+4]**

4. What are coplanar lines? Find the condition that the two lines in symmetrical form are coplanar. Also, show that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}$, $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. **[1+3+3]**

5. What are reciprocal system of vectors? Show that the scalar product of any vector of one system with a vector of the other system which does not correspond to it is zero, Find a set of vectors which form a reciprocal system to the set of vectors –i + j +k, i –j +k, i +j –k **[1+2+4]**

OR

Define scalar product of three non-zero vectors. Interpret it geometrically. Show that the position of dot and cross can be interchanged without change its value. [1+3+3]

Group "B"

[10×4=40]

6. Reduce the equation $3x^2-2xy+4y2+8x-10y+8=0$ by translating the axes into an equation with linear term missing. [4]
7. Define focal chord of a conic. In any conic, prove that the sum of reciprocals of two perpendicular focal chord is constant. [1+3]

OR

Obtain the polar equation of the conic section in the form $r = \frac{1}{1+e\cos\theta}$. [4]

8. Prove that the point $x = \frac{1(1-t^2)}{1+t^2}$ and $y = b\left(\frac{2t}{1+t^2}\right)$ is a point of an ellipse, where t is a parameter. [4]
9. Obtain the equation of plane through the intersection of the planes $x+2y+3z+4=0$ and $4x+3y+2z+1=0$ and the origin. [4]

OR

Obtain the angle between the two planes represented by $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$

10. Find the equation of the sphere having the circle $x^2+y^2+z^2=9$, $x-2y+2z=5$ as a great circle. [4]
11. Find the equation of enveloping cone of the sphere $x^2+y^2+z^2=a^2$ with vertex at the pint (α,β,γ). [4]

OR

Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y7-1=0$ having its generators parallel to the lines $x = y = z$. [4]

12. If the normal at any point P of an ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the principal planes at G_1, G_2, G_3 respectively show that $PG_1: PG_2: PG_3=a^2: b^2: c^2$ [4]
13. Prove that $[e\ m\ n]\ [a\ b\ c] = \begin{vmatrix} e.a & e.b & e.c \\ m.a & m.b & m.c \\ n.a & n.b & n.c \end{vmatrix}$ [4]
14. Show that any vector r may be expressed as $r=(r.a')\,a+(r.b')\,b+(r.c')c$ where a, b, c are three non-coplanar vectors. [4]
15. Define gradient of a scalar function and divergence of a vector function. Prove that $\text{Div}(\phi a)=\phi \text{ div } a+a.(\text{grad } \phi)$, where ϕ is a scalar function of x, y, z.

OR

Define divergence and curl of a vector function. If $\phi=\log(x^2+y^2+z^2)$, find curl (grad ϕ) [4]